1(a) $\lim_{x\to 5^-} \frac{(3x)^{1/2}}{(10-2x)^{1/2}} = +\infty$

(b) $\lim_{v\to 4} \frac{v|4-v|}{4-v}$ DNE because RHS $\neq$ LHS

$$\lim_{v\to 4^+} \frac{v|4-v|}{4-v} = \lim_{v\to 4^+} \frac{-v(4-v)}{4-v} = -4$$

$$\lim_{v\to 4^-} \frac{v|4-v|}{4-v} = \lim_{v\to 4^-} \frac{v(4-v)}{4-v} = 4.$$

(c) $\lim_{x\to 0^+} \sqrt{x}\sin\left(\frac{1}{x}\right) = 0$

Sandwich Thm: $-1 \le \sin\left(\frac{1}{x}\right) \le 1$

$$-\sqrt{x} \le \sqrt{x}\sin\left(\frac{1}{x}\right) \le \sqrt{x}$$

$$0 = \lim_{x\to 0^+}(-\sqrt{x}) \le \lim_{x\to 0^+} \sqrt{x}\sin\left(\frac{1}{x}\right) \le \lim_{x\to 0^+} \sqrt{x} = 0$$

2.(a) $y = x - \frac{1}{2x} = f(x)$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{x+h-\frac{1}{2(x+h)} - \left(x - \frac{1}{2x}\right)}{h}$$

$$= \lim_{h\to 0} \frac{h - \frac{1}{2(x+h)} + \frac{1}{2x}}{h} = \lim_{h\to 0}\left[1 + \frac{-2x+2(x+h)}{4hx(x+h)}\right]$$

$$= \lim_{h\to 0}\left[1 + \frac{2h}{4hx(x+h)}\right] = 1 + \frac{2}{4x^2} = 1 + \frac{1}{2x^2}$$

(b) $f'(3) = 1 + \frac{1}{2(9)} = \frac{19}{18}$ and $f(3) = 3 - \frac{1}{2(3)} = \frac{17}{6}$

Tangent line: $y - f(3) = f'(3)(x-3) \Rightarrow y - \frac{17}{6} = \frac{19}{18}(x-3)$

Normal line: $y - f(3) = -\frac{1}{f'(3)}(x-3) \Rightarrow y - \frac{17}{6} = -\frac{18}{19}(x-3)$

(c) $y = x - \frac{1}{2x}$ does not have a horizontal tangent. since

$\lim_{x\to +\infty}\left(x - \frac{1}{2x}\right) = +\infty$ and $\lim_{x\to -\infty}\left(x - \frac{1}{2x}\right) = -\infty$.

(d) $\lim_{x\to 0^-}\left(x - \frac{1}{2x}\right) = +\infty$ and $\lim_{x\to 0^+}\left(x - \frac{1}{2x}\right) = -\infty$

3(a) $\frac{d}{dx}(8fg) = 8f(x)g'(x) + 8f'(x)g(x)$

$$\left.\frac{d}{dx}(8fg)\right|_{x=1} = 8f(1)g'(1) + 8f'(1)g(1) = 8(1)(3) + 8(-5)(2) = \boxed{-56}$$

(b) $$\left.\frac{d}{dx}\left(\frac{f}{g}\right)\right|_{x=1} = \frac{g(1)f'(1) - f(1)g'(1)}{(g(1))^2} = \frac{2(-5)-(1)(3)}{2^2} = \boxed{-\frac{13}{4}}$$

3(c) $\frac{d}{dx}\left(\frac{g}{2f}\right)\Big|_{x=1} = \frac{1}{2}\frac{d}{dx}\left(\frac{g}{f}\right)\Big|_{x=1}$

$= \frac{1}{2}\left[\frac{f(1)g'(1) - g(1)f'(1)}{(f(1))^2}\right] = \boxed{\frac{13}{2}}$

(d) $\frac{d}{dx}(-2f+10g)\Big|_{x=1} = -2f'(1) + 10g'(1) = \boxed{40}$

4. (a) $s(t) = 24t - 3t^2$ meters

$v(t) = s'(t) = 24 - 6t$ m/s

$a(t) = s''(t) = -6$ m/s^2

(b) highest point when $v(t) = 0 \Rightarrow 24 - 6t = 0 \Rightarrow t = 4$ s

(c) height at $t=4$ is $s(4) = 24(4) - 3(4)^2 = 96 - 48 = 48$ m

(d) when $s(t) = 0$ and $t \neq 0$ we have: $24t - 3t^2 = 0$

$t(24 - 3t) = 0$

$\Rightarrow t = 0$ or $t = 8$.

rock is in the air $t = 8$ sec.

5. $y = ax^2 + bx + c$ (3 unknowns are $a, b, + c$)

(i) Passes through pt $(1,2)$: $a + b + c = 2$

(ii) Tangent to $y = x$ at the origin means $y = ax^2 + bx + c$ passes through the origin & has slope 1 when $x = 0$.

$\therefore$ $y(0) = a\cdot 0 + b\cdot 0 + c = 0 \Rightarrow c = 0$

$y'(x) = 2ax + b \Rightarrow y'(0) = 2a\cdot 0 + b = 1 \Rightarrow b = 1$

$\therefore$ $a + b + c = 2 \Rightarrow a = 2 - b - c = 2 - 1 - 0 = 1$

Summary: $\boxed{a = 1,\ b = 1,\ c = 0}$