1(a) False. Counterexample: $f(x)=x^4$, $f'(x)=4x^3$, $f''(x)=12x^2$

Note that $f'(0)=0$ and $f''(0)=0$ but $f''(x)>0$ for all $x\neq 0$.

So, $f(x)=x^4$ is always concave up $\Rightarrow$ $x=0$ is not an inflection pt.

(b) False. Newton's Method: $x_{n+1}=x_n-\dfrac{f(x_n)}{f'(x_n)}$

If $x_0=0$ then $x_1=x_0-\dfrac{f(x_0)}{f'(x_0)}=0-\dfrac{1}{3}=\boxed{-\dfrac{1}{3}}$

(c) False. If $f'(x)=g'(x)$ then $f(x)=g(x)+C$.

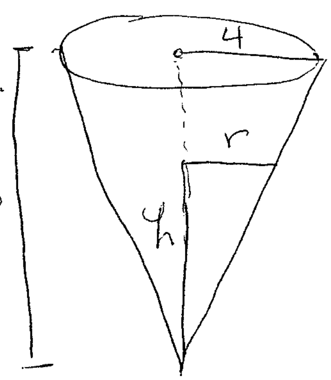
2(a) $y=\left(\dfrac{\tan\theta}{1+\sec\theta}\right)^3$ $\quad \dfrac{dy}{d\theta}=3\left(\dfrac{\tan\theta}{1+\sec\theta}\right)^2\left[\dfrac{(1+\sec\theta)\sec^2\theta-\tan^2\theta\sec\theta}{(1+\sec\theta)^2}\right]$

(b) $\lim\limits_{x\to 0}\dfrac{x^2-x-\sin 3x}{2x}=\lim\limits_{x\to 0}\left(\dfrac{x}{2}-\dfrac{1}{2}-\dfrac{3}{2}\left(\dfrac{\sin 3x}{3x}\right)\right)=-\dfrac{1}{2}-\dfrac{3}{2}=\boxed{-2}$

(c) $x+\sin y=xy \Rightarrow 1+\cos y\dfrac{dy}{dx}=y+x\dfrac{dy}{dx}$

$\Rightarrow \dfrac{dy}{dx}(\cos y-x)=y-1 \Rightarrow \boxed{\dfrac{dy}{dx}=\dfrac{y-1}{\cos y-x}}$

3.



Given: $\dfrac{dV}{dt}=-5\ \text{ft}^3/\text{min}$

Find: $\dfrac{dh}{dt}$ when $h=6$ ft.

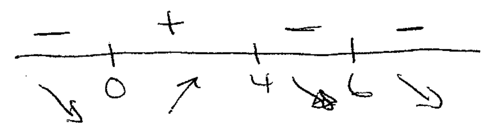
$V=\frac{1}{3}\pi r^2 h$. We know $\dfrac{h}{r}=\dfrac{10}{4} \Rightarrow r=\dfrac{2}{5}h$

$\therefore V=\dfrac{4}{75}\pi h^3$

$\dfrac{dV}{dt}=\dfrac{dV}{dh}\dfrac{dh}{dt}=\dfrac{4}{25}\pi h^2\dfrac{dh}{dt} \Rightarrow \dfrac{dh}{dt}=\dfrac{1}{\frac{4}{25}\pi h^2}\dfrac{dV}{dt}$

$\Rightarrow \left.\dfrac{dh}{dt}\right|_{h=6}=\dfrac{1}{\frac{4}{25}\pi(36)}\cdot(-5)=-\dfrac{125}{144\pi}$ ft/min

4. $f'(x)=\dfrac{4-x}{x^{1/3}(6-x)^{2/3}}$ Critical pts: $x=0,4,6$



$f''(x)=\dfrac{-8}{x^{4/3}(6-x)^{5/3}}$. Critical pts: $x=0,6$

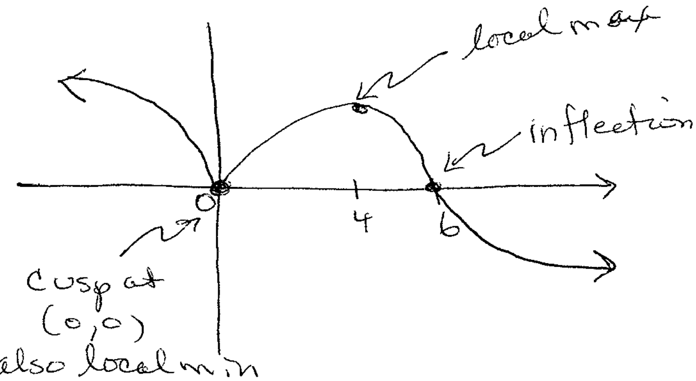
(a) increasing: $(0,4)$

(b) local min at $x=0$
local max at $x=4$

(c) concave down: $(-\infty,0)\cup(0,6)$

(d) $x=6$ is an inflection pt

(e)



5. Estimate $(26)^{1/3}$.

Let $f(x) = x^{1/3}$. Then $f'(x) = \frac{1}{3}x^{-2/3}$

Linearization of $f(x) = x^{1/3}$ is given by:

$L(x) = f(a) + (x-a) f'(a)$ (choose $a = 27$ so $f(a) = (27)^{1/3} = 3$

also, $f'(27) = \frac{1}{3}\frac{1}{(27)^{2/3}}$

$= \frac{1}{3}\frac{1}{(3^3)^{2/3}}$

$L(x) = 3 + (x-27)\frac{1}{3}\frac{1}{(27)^{2/3}}$

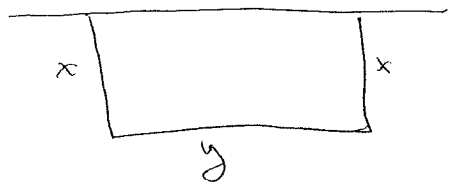
$= 3 + \frac{1}{27}(x-27)$

$= 3 + \frac{1}{27}x - 1 = 2 + \frac{1}{27}x$

$L(26) = 2 + \frac{26}{27} = \frac{80}{27}$

Yes, this is a good approximation because $\sqrt[3]{26} \approx \sqrt[3]{27} = 3$.
Note that $\frac{80}{27}$ is just a bit less than 3, as expected.

6. <u>Given</u>: Area $= 180 m^2 = xy$



$P = 2x + y$

<u>Want</u>: minimize perimeter P.

$180 = xy \Rightarrow y = \frac{180}{x}$

$\therefore P(x) = 2x + \frac{180}{x}$, $x > 0$

$P'(x) = 2 - \frac{180}{x^2}$ $P'(x) = 0$ when $2 - \frac{180}{x^2} = 0$

$\Rightarrow x^2 = 90$

$\Rightarrow x = \pm\sqrt{90}$ (discard $x = -\sqrt{90}$ since not physically significant

$P''(x) = \frac{360}{x^3} > 0$ for all $x > 0$

$\therefore x = \sqrt{90}$ is a min.

Dimensions: $x = \sqrt{90} = 3\sqrt{10}$

$y = \frac{180}{\sqrt{90}} = 6\sqrt{10}$