APPM 1350 - Exam 3 Makeup Solutions

1(a). $\int \frac{\sec^2(\sqrt{x}+1)}{\sqrt{x}}\,dx = 2\int \sec^2 u\,du$ Let $u=\sqrt{x}+1$, $du = \frac{1}{2}x^{-1/2}dx$

$= 2\tan u + C$

$= 2\tan(\sqrt{x}+1) + C$

(b) $\int_0^1 \frac{1}{\sqrt[3]{8-7x}}\,dx = -\frac{1}{7}\int_8^1 u^{-1/3}\,du$ Let $u = 8-7x$, $du = -7dx$

$= \frac{1}{7}\int_1^8 u^{-1/3}du = \frac{1}{7}\cdot\frac{3}{2}u^{2/3}\Big|_1^8 = \frac{3}{14}\left[8^{2/3} - 1^{2/3}\right] = \frac{3}{14}[4-1] = \boxed{\frac{9}{14}}$

(c) $\int_0^{\pi/4} \frac{\sin x}{\cos^2 x}\,dx = -\int_1^{\sqrt{2}/2} \frac{1}{u^2}\,du$ Let $u = \cos x$, $du = -\sin x\,dx$

$= -\frac{u^{-1}}{-1}\Big|_1^{\sqrt{2}/2} = \frac{1}{u}\Big|_1^{\sqrt{2}/2} = \frac{2}{\sqrt{2}} - 1 = \boxed{\sqrt{2}-1}$

2.(a) FTC - in your text.

(b) $F(x) = \int_1^{x^2} \frac{1}{t}\,dt$. $F'(x) = \frac{1}{x^2}\cdot 2x = \frac{2}{x}$ $\Rightarrow F'(7) = \boxed{\frac{2}{7}}$

(c) $\lim_{n\to\infty} \sum_{k=1}^{n} \left(1 + 9\frac{k^2}{n^2}\right)\cdot\frac{1}{n} = \int_0^1 (1+9x^2)\,dx = x + \frac{9x^3}{3}\Big|_0^1 = 4$

(d) $\int_0^2 \sin(x^2)\,dx$ n subintervals of $[0,2]$, each of length

$\Delta x = \frac{b-a}{n} = \frac{2}{n}$

right hand endpt: $x_k = k\cdot\Delta x = \frac{2k}{n}$

$f(x) = \sin(x^2)$ so $f(x_k) = \sin\left(\frac{4k^2}{n^2}\right)$

$\therefore \int_0^2 \sin(x^2)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} \underbrace{\sin\left(\frac{4k^2}{n^2}\right)}_{f(x_k)}\cdot\underbrace{\frac{2}{n}}_{\Delta x}$

3. Given: $a(t) = -20$ ft/sec^2

$v(t) = -20t + C_1$ Since $v(0) = 100$ ft/sec, we have

$v(0) = -20\cdot 0 + C_1 = 100$

$\Longrightarrow C_1 = 100$

$s(t) = \int v(t)\,dt = \int(-20t+100)\,dt$

$= -10t^2 + 100t + C_2$

$s(0) = 0 \Rightarrow C_2 = 0$.

$\therefore s(t) = -10t^2 + 100t$

(a) Car stops when $v=0$.

$\Rightarrow v(t) = -20t + 100 = 0$

has $\boxed{t = 5 \text{ sec}}$

(b) In 5 sec, car travels

$s(5) = -10(5)^2 + 100(5)$

$= \boxed{250 \text{ ft}}$

4 (a) $f(x) = \ln(x-3)$

$D_f = (3, \infty)$

$R_f = (-\infty, \infty)$

(b) $f'(x) = \frac{1}{x-3}$. Since $\frac{1}{x-3} > 0$ for all $x > 0$, we know $f(x) = \ln(x-3)$ is increasing on its domain.

$\therefore$ f is 1-1 and hence invertible

(c) $y = \ln(x-3)$

$\cancel{x} = \ln(y-3)$

$e^x = y - 3$

$y = e^x + 3$

$f^{-1}(x) = e^x + 3$

(d) $D_{f^{-1}} = (-\infty, \infty)$

$R_{f^{-1}} = (3, \infty)$

5. $g(x) = x^2 + \int_{3x}^{-3} \frac{1}{t^2+2}\,dt = x^2 - \int_{-3}^{3x} \frac{1}{t^2+2}\,dt$

(a) Find $L(x)$ at $x = -1$.

$g'(x) = 2x - \frac{1}{(3x)^2+2} \cdot 3 = 2x - \frac{3}{9x^2+2}$

$g'(-1) = 2(-1) - \frac{3}{9(-1)^2+2} = -2 - \frac{3}{11} = \frac{-25}{11}$

$g(-1) = (-1)^2 - \int_{-3}^{-3} \frac{1}{t^2+2}\,dt = 1$

$L(x) = g(-1) + g'(-1)(x-(-1))$

$L(x) = 1 - \frac{25}{11}(\cancel{x}+1)$

(b) $L(-1.1) = 1 - \frac{25}{11}(-1.1+1) = 1 - \frac{25}{11} \cdot \frac{1}{10} = 1 - \frac{25}{110} = \frac{85}{110} = \boxed{\frac{17}{22}}$