This handout may be used as a guide to review and prepare for Exam 2. However, it may not be comprehensive and complete and hence you should use it only as a supplement to the book and the information provided in the course website.

1 Main Concepts

1.1 Chain Rule and Implicit Differentiation

1.2 Application of derivatives: example Rates of change

1.3 Mean Value Theorem, Rolle's Theorem: Statement and Application

1.4 Extreme Values, Asymptotes, Sketching the function

1.5 Optimization (i.e. Word problems on maxima and minima)

1.6 Linearization

1.7 Newton's Method

2 Notes and sample problems

2.1 Chain Rule and Implicit Differentiation

1. **The Chain Rule** (snapshot): $(f \circ g)'(x) = \frac{d}{dx}\Bigg(f(g(x))\Bigg) = f'(g(x)).g'(x)$

   sample problems:

   (a) Find the derivative of $g(t) = \tan(5 - \sin 2t)$. (page 157, example 5)

   (b) Find the tangent and normal to the curve $x^2 - xy + y^2 = 7$ at the point $(-1, 2)$. (page 167, example 4)

   (c) Find $\frac{df}{dt}$, if $f(t) = \frac{1}{6}\Bigg(1 + \cos^2 7t\Bigg)^3$. (Exam 2, Fall 08)

   (d) Find $\frac{d^2z}{dy^2}$ for the function $6z^3y + 4y = 1$. (Exam 2, Fall 08)

2.2 Application of derivatives: example Rates of change

sample problems:

1. At time t, the position of a body moving along the s-axis is $s = t^3 - 6t^2 + 9t$. (1) Find its acceleration each time the velocity is 0. (2) When is the body moving forward and when is it moving backward? (3) When is the body's velocity increasing ? (worksheet 5, Fall 09).

2. A cop is trying to videotape a robber who has stolen a Porche. Unknown to him, there is an automatic camera placed on a pole 132 ft above the ground tracking his movements. The car is moving directly towards the pole at 264 ft/sec. What would the speed of the car with respect to the camera be when the car is 30 ft from the base of the pole ? (Forgiveness midterm 2, Spr 07)

3. The thin lens equation in optics is

$$\frac{1}{D} + \frac{1}{S} = \frac{1}{F}$$

   where D is the distance from the object to the lens, S is the distance from the imge to the lens, and F is the focal length of the lens. Let a F = 6 cm for a specific lens and suppose an object is moving towards the lens at the rate of 2 cm/sec. How fast is the image distance S changing at the moment when the distance between the object and the lens is D = 10 cm? (Exam 2, Fall 05).

2.3 Mean Value Theorem, Rolle's Theorem: Statement and Application

1. **Rolle's Theorem:** Let y = f(x) be a continuous function in [a,b] and differentiable in (a,b) (hypothesis). If f(a) = f(b) = 0, then there exists atleast a point $c \in (a, b)$ where f'(c) = 0.

2. **Mean Value theorem:** Given the same hypothesis as in the above theorem, there exists atleast a point $c \in (a, b)$ where $f'(c) = \frac{f(b)-f(a)}{b-a}$.

   sample problems:

   (a) State true or false and justify your answer: The function $g(x) = \frac{\sin x}{x}$ on $[\pi/4, \pi/2]$ satisfies the hypothesis of the Mean Value Theorem.(Spr 03).

   (b) It took 14 sec for a thermometer to rise from -19^oC to 100^oC when it was taken from a freezer and put into boiling water. Show that somewhere along the way the mercury was rising exactly at $8.5^oC/sec$.(Worsheet 7, fall 09).

2.4 Extreme Values, Asymptotes, Sketching the function

1. (a) Review the definition of critical points, points of inflexion, concavity, asymptotes.
   (b) Review the technique to find critical points based on first derivative and how to find if the function is increasing or decreasing based on how the sign of first derivative changes.
   (c) Review the technique to find inflexion points and how to find regions of concave up and concave down based on how the sign of the second derivative changes in different regions of the domain.
   (d) Review how to find vertical, horizontal and oblique asymptotes (Long division ?).
   (e) Practise sketching curves based on the above information.

sample problems:

1. Let $f(x) = \frac{x^3+1}{x^2}$.
   (a) Is f(x) odd, even ot neither?
   (b) Does f(x) have an oblique asymptote? If yes, what is it ?
   (c) Does f(x) have horizontal and vertical asymptotes ? If yes, what are they ?
   (d) Where is f(x) increasing and where is it decreasing ?
   (e) In which interval(s) is f(x) concave up and concave down? (Spr 05)
2. Graph the function $f(x) = \frac{x^3-1}{x^2-1}$. (*removable discontinuity*)(page 226, example 10).
3. Find asymptotes of the curve $y = 2 + \frac{\sin x}{x}$ (*Sandwich theorem*)(page 226, example 11).
4. Find the asymptotes and all other necessary information to graph the function $y = \frac{x^3-2x^2+x+1}{x-x^2}$. (page 232, ex: 92).
5. Let $f(x) = \frac{x^2+3}{x-1}$, $x \neq 1$. (1) Findthe maxima and minima. (2) Deteremine potential horizontal, vertical and oblique assymptotes. (3) Sketch the graph and label it.(midterm 2, Spr 09)

2.5 Optimization (i.e. Word problems on maxima and minima)

Please practise a lot of problems from this section as students generally struggle a lot to interpret the wording of optimization problems in the test. Note that some problems may require use of dummy variables which may not be stated/hinted at in the question.

sample problems:

1. A window is in the form of a rectangle surmounted by a semi-circle. The rectangle is of clear glass while the semicircle is of tinted glass that transmits only half as much light per unit area as clear glass does. The total perimeter is fixed. Find the proportion of the window that will admit the most light. Neglect the thickness of the frame. (page 244, ex: 25, HW problem)
2. The USP service will accept a box for domestic shipment only if the sum of its length and girth doesnot exceed 108 in. What dimensions will give a box with a square end the largest possible volume? (Exam 2, Fall 06)
3. Assuming that the petrol burnt per hour in driving a motor boat varies as the cube of its velocity, show that the most economical speed when going against a current μ m.p.h. is 1.5μ m.p.h.
4. Find the altitude and the semi-vertical angle of a cone of least volume which can be circumscribed to a sphere of radius α.
5. In the first quadrant, find the coordinates of the point on the curve $y = \frac{1}{x}$ that is closest to the origin. (Spr 01).

2.6 Linearization

1. **Definition:** If f is differentiable at x = a, then the linearization of f at a is $L(x) = f(a) + f'(a)(x - a)$.
2. **differential:** Let $y = f(x)$ be a differentiable function. The differential dx is an independent variable. The differential dy is given by $dy = f'(x)dx$.

sample problems:

1. Find the linearization of the following curves and then graph the curve and the linearization together. (1) $f(x) = \sec x$ at $x = 0$ and $-\pi/3$ (2) $f(x) = \tan x$ at $x = 0$ and $x = \pi/4$.(page 258, ex: 15,16)

2.7 Newton's Method

1. **iteration:** $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$.

sample problems:

1. Find the x-coordinate of the point where the curve $y = x^3 - x$ croses the horizontal line y = 1.(page 262, example 2).