

On the front of your bluebook, please write: a grading key, your name, student ID, and section and instructor. This exam is worth 100 points and has 5 questions. Show all work! Answers with no justification will receive no points. Please begin each problem on a new page. No notes, calculators, or electronic devices are permitted.

1. (15 points) Find the requested limits. JUSTIFY each of your answers.

(a) $\lim_{x \rightarrow 1} \left[\frac{1}{x-1} - \frac{1}{x^2 - 3x + 2} \right]$

(b) $\lim_{v \rightarrow 4^+} \frac{4-v}{|4-v|}$

(c) $\lim_{x \rightarrow 0} f(x)$ where $f(x) = \begin{cases} x^2 \cos(1/x^2) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

2. (20 points) Let $f(x) = \frac{x^2 - 3x + 2}{(x^2 - 1)(x - 2)}$

- (a) Using interval or set notation, give the domain of f .
(b) Graph the function $f(x)$. Be sure to label your graph carefully. (Hint: Simplify $f(x)$ algebraically before graphing!)
(c) Give the definition for a function $f(x)$ to be continuous at a point c .
(d) Find the points x where $f(x)$ is discontinuous and show which part of the Continuity Test fails.
(e) Using interval or set notation, write down the set of x where $f(x)$ is continuous.

3. (20 points) Let $y = \sqrt{2x - 1}$.

- (a) Calculate dy/dx using the definition of the derivative.
(b) Find the equation of the tangent and normal lines to the curve at $x = 5$.
(c) Does any tangent to the curve $y = \sqrt{2x - 1}$ pass through the origin? If so, find this tangent. If not, why not?

4. (15 points) Find the requested quantities.

(a) Find dy/dx for $y = x^3 + 3x^{-2} - 4\sqrt{x} + \pi$.

(b) Find $g'(1)$ for $g(x) = (x^3 - x^2 f(x))(2 - \frac{1}{x})$ when $f(1) = 2$ and $f'(1) = 5$.

(c) Find dy/dx for $y = \frac{x^2 + 3}{5 - 2x^3}$.

5. (30 points) True or False. If the statement is true, write out the word TRUE and explain why it is true. If the statement is false, write the word FALSE and either correct the statement to make it true or give a counterexample to demonstrate why it is false.

- (a) The point $(1, 0)$ is outside of the circle of radius 3 centered at $(-1, 2)$.
(b) If $f(x)$ is any function with $f(a) < 0$ and $f(b) > 0$, then there is some value c , between a and b with $f(c) = 0$.
(c) If $f(x)$ is continuous at 3, then it is differentiable at 3.
(d) $\frac{d}{dx}(fg) = f'g'$
(e) $\frac{d}{dx}|x^2 + x| = |2x + 1|$
(f) If $f'(r)$ exists, then $\lim_{x \rightarrow r} f(x) = f(r)$.