APPM 1350 **Midterm #2** **Fall 2009**

On the front of your bluebook, please write: a grading key, your name, student ID, and section and instructor. This exam is worth 100 points and has 6 questions. **Show all work!** Answers with no justification will receive no points. Please begin each problem on a new page. No notes, calculators, or electronic devices are permitted.

1. (15 points) True or False. If the statement is true, write out the word TRUE and *explain why it is true.* If the statement is false, write the word FALSE *explain why it is false.*

   (a) Suppose $y = f(x)$ has a critical point at $x = c$. Then $f'(c) = 0$.

   (b) Let $f(x) = x^3 - 4x + 2$ If we were to use Newton's Method to estimate the zero of $f(x)$ with $x_0 = 0$ then $x_1 = -1/2$.

   (c) Suppose both $f(x)$ and $g(x)$ are twice differentiable functions with a local minimum at the point $x = c$. If $f(c) < 0$ and $g(c) < 0$ then $h(x) = f(x)g(x)$ has a local maximum at $x = c$.

2. (15 points) Find the requested information.

   (a) Find $\dfrac{dy}{d\theta}$ when $y = \sin\sqrt{\theta}\tan(\frac{1}{\theta})$. (Do not simplify your answer.)

   (b) $\displaystyle\lim_{x\to 0}\frac{\tan(3x)}{\sin(8x)}$

   (c) Find $\dfrac{dy}{dx}$ in the equation $x + \tan(xy) = 0$.

3. (20 points) A girl flies a kite at a height of 300 ft, the wind carrying the kite horizontally away from her at a rate of 25 ft/sec. How fast is the angle changing at the moment that the horizontal distance between the girl and the kite is 400 ft?

4. (20 points) Suppose you know that $f'(x) = (x-2)^2(x+3)^2$ and that $f(0) = 2$. Answer the following questions about the original function $f(x)$.

   (a) On what interval(s) is $f(x)$ increasing?

   (b) Give the x coordinate of any local maxima or minima of $f(x)$ that might exist.

   (c) On what interval(s) is $f(x)$ concave down?

   (d) Give the x coordinate of any inflection point.

   (e) Sketch the graph $y = f(x)$.

5. (10 points) Let $f(x) = \dfrac{x^2 - 49}{x^2 + 5x - 14}$. Using the definitions, find all vertical and horizontal asymptotes of $f(x)$.

6. (20 points) Motor oil cans used to be cylindrical with cardboard sides and metal top and bottom. Such a can is to be constructed with a volume of 1 liter. If the metal costs three times as much per square centimeter as the cardboard, what dimentions should the can have to minimize the cost? Assume that cardboard costs 1 cent per cm^2. (Hint: 1 liter = 1000 cm^3)

EXTRA CREDIT (5 points) Suppose that f and g are differentiable on $[a, b]$ and that $f(a) = g(a)$ and $f(b) = g(b)$. Show that there is at least one point between a and b where the tangents to the graphs of f and g are parallel.