

On the front of your bluebook, please write: a grading key, your name, student ID, and section and instructor. This exam is worth 100 points and has 5 questions.

- Show all work! Answers with no justification will receive no points.
- Please begin each problem on a new page.
- No notes, calculators, or electronic devices are permitted.

1. (21 points) Evaluate the following integrals.

(a) $\int_0^1 \frac{3u}{1+u^2} du$

(b) $\int \sin^2(6\theta + 4) d\theta$

(Hint: You may find the following identity useful: $\cos(2\theta) = 1 - 2\sin^2(\theta)$.)

(c) $\int_0^\pi e^{\sec(2t)} \sec(2t) \tan(2t) dt$

2. (28 points) Find the requested information:

(a) Evaluate $\sum_{j=1}^n (2j + 3)$. Your final answer should be in terms of n .

(b) Use logarithmic differentiation to find $\frac{dy}{dx}$ for

$$y = \frac{(3x+5)^{\frac{10}{3}} (2x^2+9)^{\frac{3}{2}}}{(5x-4)^{\frac{7}{5}}}. \text{ Find the expression for } \frac{dy}{dx} \text{ but do not simplify your answer further.}$$

(c) Find the total area between the curve $y = 4x^3 - 4$ and the x-axis on the interval $[0, 2]$.

(d) Find the average value of $g(x) = 4x^3 - 4$ on the interval $[0, 2]$.

3. (14 points)

(a) Carefully state the Fundamental Theorem of Calculus, parts 1 and 2. Be sure to include the hypotheses of the theorem in your answer.

(b) Find the linearization at $x = 1$ for $f(x) = 4 + 2 \int_2^{x^2+1} \frac{1}{2+t} dt$.

4. (21 points) Estimate the area under the curve $f(x) = x^2$ on the interval $[1, 4]$

(a) using a Riemann sum. Use a partition with three subintervals of equal length, and evaluate $f(x)$ at the left end points of your subintervals.

(b) using the trapezoidal rule with three intervals.

(c) Use E_T to give an upper bound for the error in the trapezoidal rule in part 4(b).

5. (16 points)

(a) Show that the function

$$f(x) = e^{3x+1} - 2$$

is invertible for $x \in (-\infty, \infty)$.

(b) Find the inverse, $f^{-1}(x)$, for $f(x)$ in part (a).

(c) What is the domain and range for $f^{-1}(x)$?