1(a) $\int_0^1 \frac{3u}{1+u^2}\,du = \int_1^2 \frac{3}{2t}\,dt = \frac{3}{2}\ln t\Big|_1^2 = \frac{3}{2}(\ln 2 - \ln 1) = \boxed{\frac{3}{2}\ln 2}$

Let $t = 1+u^2$ $\quad u=0 \Rightarrow t=1$
$dt = 2u\,du$ $\quad u=1 \Rightarrow t=2$

(b) $\int \sin^2(6\theta+4)\,d\theta = \frac{1}{6}\int \sin^2 u\,du = \frac{1}{12}\int (1-\cos 2u)\,du$

Let $u = 6\theta+4$
$du = 6\,d\theta$

identity $\sin^2 u = \frac{1-\cos 2u}{2}$

$= \frac{1}{12}\left[u - \frac{1}{2}\sin(2u)\right] + C$

$= \frac{1}{12}\left[6\theta + 4 - \frac{1}{2}\sin(12\theta+8)\right] + C$

$= \boxed{\frac{\theta}{2} + \frac{1}{3} - \frac{1}{24}\sin(12\theta+8) + C} = \boxed{\frac{\theta}{2} - \frac{1}{24}\sin(12\theta+8) + C'}$

$C'$ is a different constant than $C$.

(c) $\int_0^\pi e^{\sec(2t)}\sec(2t)\tan(2t)\,dt$.

<u>Note:</u> $\sec(2t) = \frac{1}{\cos 2t}$ and $\tan(2t) = \frac{\sin(2t)}{\cos(2t)}$. Further, $\cos(2t)=0$ when $t = \frac{\pi}{4}$ and $\frac{3\pi}{4}$ and the integrand is not defined at these values of $t$. Hence, the integral does not exist! (This is the same reason that $\int_{-1}^1 \frac{1}{t}\,dt$ does not exist.)

However, if you did the u-substitution $u = \sec(2t)$
$du = 2\sec(2t)\tan(2t)\,dt$

to obtain $\int_1^1 \frac{1}{2}e^u\,du = 0$ you received full credit.

2(a) $\sum_{j=1}^n (2j+3) = \sum_{j=1}^n (2j) + \sum_{j=1}^n 3 = 2\sum_{j=1}^n j + \sum_{j=1}^n 3 = 2\frac{n(n+1)}{2} + 3n$
$= \boxed{n^2+4n}$

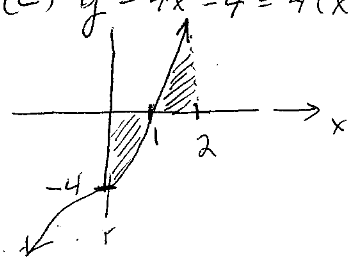
(b) $y = \frac{(3x+5)^{10/3}(2x^2+9)^{3/2}}{(5x-4)^{7/5}}$

$\ln y = \frac{10}{3}\ln(3x+5) + \frac{3}{2}\ln(2x^2+9) - \frac{7}{5}\ln(5x-4)$

$\frac{1}{y}\frac{dy}{dx} = \frac{10}{3}\cdot\frac{3}{3x+5} + \frac{3}{2}\,\frac{4x}{2x^2+9} - \frac{7}{5}\cdot\frac{5}{5x-4}$

$\Rightarrow \frac{dy}{dx} = \frac{(3x+5)^{10/3}(2x^2+9)^{3/2}}{(5x-4)^{7/5}}\left[\frac{10}{3x+5} + \frac{6x}{2x^2+9} - \frac{7}{5x-4}\right]$.

(c) $y = 4x^3-4 = 4(x^3-1) = 4(x-1)(x^2+x+1)$. and $x^2+x+1$ does not factor further.




From graph, we see:

Area $= \int_0^2 |4x^3-4|\,dx = \int_0^1 |4x^3-4|\,dx + \int_1^2 |4x^3-4|\,dx$

$= \int_0^1 (4-4x^3)\,dx + \int_1^2 (4x^3-4)\,dx = \boxed{14}$

2(d) Average value of $f = \frac{1}{2-0}\int_0^2 (4x^3-4)\,dx$

$= \frac{1}{2}(x^4-4x)\Big|_0^2 = \frac{1}{2}(16-8) = \boxed{4}$

3(a) See text for complete statement (p. 333 and p. 336)

(b) $f(x) = 4 + 2\int_2^{x^2+1} \frac{1}{2+t}\,dt$ $\qquad f(1) = 4 + 2\int_2^2 \frac{1}{2+t}\,dt = 4$

$f'(x) = 2\cdot\frac{1}{2+(x^2+1)}\cdot 2x = \frac{4x}{3+x^2}$ $\qquad f'(1) = \frac{4}{3+1} = 1$

$\therefore L(x) = f(1) + f'(1)(x-1) = 4 + 1(x-1) = x+3$

$\boxed{L(x) = x+3}$

4. $f(x) = x^2$ on $[1, 4]$

(a) $n=3$, $\Delta x = \frac{b-a}{n} = \frac{4-1}{3} = 1$

Riemann sum (left-hand endpts) $= \Delta x\,(f(1)+f(2)+f(3)) = 1 + 2^2 + 3^2 = \boxed{14}$

(b) $T = \frac{h}{2}\left[f(1) + 2f(2) + 2f(3) + f(4)\right] = \frac{1}{2}\left[1 + 2\cdot 2^2 + 2\cdot 3^2 + 4^2\right] = \boxed{\frac{43}{2}}$

(c) $|E_T| \le \frac{b-a}{12} h^2\cdot M$ where $M = \max_{a\le x\le b} |f''(x)|$

Here, $f(x) = x^2 \Rightarrow f'(x) = 2x \Rightarrow f''(x) = 2$ $\quad\therefore M = 2$

$|E_T| \le \frac{4-1}{12}\cdot(1)^2\cdot 2 = \frac{1}{2} \Rightarrow \boxed{|E_T| \le \frac{1}{2}}$

5(a) $f(x) = e^{3x+1} - 2$

$f'(x) = 3e^{3x+1} > 0$ for every $x \in (-\infty, \infty)$

$\therefore f(x)$ is monotone increasing $\Rightarrow f$ is 1-1 $\Rightarrow f$ is invertible

(b) $y = e^{3x+1} - 2$

$x = e^{3y+1} - 2$ (interchange $x$ + $y$)

$x+2 = e^{3y+1}$

$\ln(x+2) = \ln(e^{3y+1}) = 3y+1$

$y = \frac{\ln(x+2) - 1}{3} \Rightarrow f^{-1}(x) = \frac{\ln(x+2)-1}{3}$

(c) Domain of $f^{-1}$: $(-2, +\infty)$ since we must have $x+2>0$

Range of $f^{-1}$: $(-\infty, \infty)$ (same as domain of $f$)