Calculus I
Review for midterm#3

November 16, 2009

Show all your work.

1. Evaluate the following integrals:

(a) $\int \frac{\sqrt[3]{v}-1}{\sqrt{v}}dv$

(b) $\int e^t \sin(e^t - 2)\,dt$

(c) $\int x^{1/3}\sin\left(x^{4/3}-8\right)dx$

(d) $\int \left(\sec^2(t) - \frac{1}{\sqrt[5]{t^3}} + \pi\right)dt$

(e) $\int 10t\sqrt{2t+5}dt$

(f) $\int \frac{(2t+1)^2}{\sqrt{t+3}}dt$

(g) $\int \frac{1}{\sqrt{x}\left(1+\sqrt{x}\right)^2}dx$

(h) $\int \cos(\sin(\sin(x)))\cos(\sin(x))\cos(x)\,dx$

(i) $\int \frac{\pi}{2}\cos(x)\sin(\pi+\pi\sin(x))\,dx$

(j) $\int \frac{dx}{\sqrt{5x+8}}$

(k) $\int \frac{\cos^3(x)+1}{\cos^2(x)}dx$

(l) $\int \sin^2(t)\,dt$

(m) $\int \cos^5\left(\frac{\theta}{2}\right)d\theta$

(n) $\int \csc^2\sqrt{x}\sqrt{\frac{\tan\sqrt{x}}{x}}dx$

(o) $\int \sec^2(x)\sqrt{1+\sqrt{\tan(x)}}dx$

(p) $\int \frac{x^2-x}{x+1}dx$

(q) $\int e^{\csc(x)}\csc(x)\cot(x)\,dx$

(r) $\int \left(w^{1/3}-2\right)\left(w^{2/3}+1\right)dw$

(s) $\int \frac{\cos\sqrt{x}}{\sqrt{x}\sin^2\left(\sqrt{x}\right)}dx$

(t) $\int \left(x^3+x+1\right)e^{\left(x^4+2x^2+4x\right)}dx$

(u) $\int \frac{e^x+e^{-x}}{(e^x-e^{-x})^{2009}}dx$

(v) $\int \frac{e^{\sqrt{2y+1}}}{\sqrt{2y+1}}dy$

(w) $\int \frac{\sec^2(\theta)}{3-2\tan(\theta)}d\theta$

(x) $\int \frac{1}{x^{10}+x}dx$

2. Evalutae the following definite integrals:

(a) $\int_{\pi/3}^{\pi/2}\sin(\theta)\cos^3(\theta)\,d\theta$

(b) $\int_0^{\pi/4}\tan(x)\,dx$

(c) $\int_{\frac{\pi^2}{4}}^{\pi^2}\frac{\cos\left(\sqrt{x}\right)}{\sqrt{x}}dx$

(d) $\int_0^{\pi}\sin^2\left(\frac{z}{2}\right)\cos\left(\frac{z}{2}\right)dz$

(e) $\int_0^1 r^2\sqrt{1-r^3}dr$

(f) $\int_0^{ln3} e^x\left(1+e^x\right)^{1/2}dx$

(g) $\int_2^4 \frac{dx}{x(\ln(x))^2}$

(h) $\int_1^4 \frac{1}{\sqrt{x}\left(1+\sqrt{x}\right)^2}dx$

(i) $\int_{-3}^{3} r\sqrt{r^2+1}dr$

(j) $\int_{-\sqrt{3}}^{\sqrt{3}}\frac{4x}{\sqrt{x^{10}+1}}dx$

(k) $\int_{-1}^{1}\cos\left(x^2\right)2x\,dx$

(l) $\int_{-2}^{-1}\frac{3x^4-4x^2}{x^2}dx$

(m) $\int_0^2 f(x)\,dx$, where $f(x) = \begin{cases} \sin\left(\frac{\pi}{2}x\right) & , x\le 1 \\ 1 & , x>1 \end{cases}$

(n) $\int_{-ln3}^{ln3}\frac{e^x}{e^x+4}dx$

(o) $\int_1^4 \frac{1}{\sqrt{x\sqrt{1+\sqrt{x}}}}dx$

(p) $\int_0^{13}\frac{1}{\sqrt[3]{(1+2x)^2}}dx$

(q) $\int_0^a x^3\sqrt{a^2-x^2}dx$

(r) $\int_0^{\sqrt{3}}\frac{4x}{\sqrt{x^2+1}}dx$

(s) $\int_4^9 \frac{1}{\sqrt{z}}\left(\frac{1-\sqrt{z}}{1-z}\right)^2 dz$

(t) $\int_1^{\sqrt{2}} xe^{-x^2}dx$

(u) Given $\int_3^{-1}[1+f(x)]\,dx = -1$,and $\int_{-1}^{5}\left[2f(x)-\frac{1}{6}\right]dx = 11$, find $\int_3^5 f(x)\,dx$.

3. Evaluate the following expressions:

(a) $\sum_{k=1}^{n}\left(2+(k-1)^3\right)$

(b) $\sum_{k=-2}^{3}\sin\left(\frac{k\pi}{3}\right)\cos(k\pi)$

(c) $\sum_{k=1}^{3}\frac{k}{k^2+1}$

(d) Find the domain of the function $f(x) = \ln\left(x^2-1\right)$.

4. **Always True/ False.**

(a) There exists no function $f(x)$ such that $f(x) = f^{-1}(x)$.

(b) For some c in $[a, b]$, $\frac{1}{b-a}\int_a^b f(x)\,dx = f(c)$.

(c) If $f'(x) = g'(x)$ on the interval $[a, b]$, then $f(b) - f(a) = g(b) - g(a)$.

(d) $6 \leq \int_0^3 \sqrt{4 + x^2}dx \leq 3\sqrt{13}$.

(e) $|\int_a^b f(x)\,dx|$ will find the area between the curve and the x-axis on the interval [a, b].

(f) If $f(x)$is odd, then $\int_0^x f(t)^2\,dt$ is odd.

(g) If $f(x) = x^n$, where n is an odd integer, then $f^{-1}(x)$ exists for all values of x.

(h) If $\int_0^2 f(x)\,dx = M$ and $\int_0^2 g(x)\,dx = N$, then $\int_0^2 f(x) \cdot g(x)\,dx = M \cdot N$.

(i) The trapezoidal rule overestimates a denite integral if the integrand is positive and concave up over the interval of integration.

(j) If $f(x) = f(-x)$ over the interval $[-a, a]$, then $\int_{-a}^a f(x)\,dx = 0$.

(k) $\ln|\tan(x)| = -\ln|\cot(x)|$

(l) $\int_{-1}^1 \frac{1}{x^2}dx = -2$

(m) $f(x) = x^2 + x$, then $f^{-1}(x) = \frac{1+\sqrt{1+4x}}{2}$.

(n) $\frac{d}{dx}\left[\int_0^1 \frac{1+t^2}{1+t^3}dt\right] = \frac{1+x^2}{1+x^3}$, for all $0 < x < 1$.

(o) If $|f(x)| \geq |g(x)|$ on $[a, b]$, then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$.

(p) $f\left(f^{-1}(x)\right) = 1$

5. Find a curve $y = f(x)$ with the following two properties:

   (a) $\frac{d^2y}{dx^2} = 6x$

   (b) Its graph passes through the point $(0, 1)$ and has a horizontal tangent there.

6.

   (a) Evaluate the following limit by identifying it with a definite integral $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n+k}$.

   (b) Find $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k^4}{n^5}$ by recognizing that the expression equals a definite integral and then evaluating the integral.

7. [Fall 2006] Consider $f(x) = 2x - x^2$ on the interval [0,3]

   (a) Sketch the graph of the curve $y = f(x)$.

   (b) What is the average value of $f(x) = 2x - x^2$ on [0,3]?

   (c) At what value or values of x in [0,3] is the average value of $f(x)$ attained?

8. [Spring 2009] Consider the function $f(x) = \frac{1}{\pi}\sqrt{4 - x^2}$.

   (a) Find the average value of the function f on the interval [0, 2]. (Hint: Interpret the integral as an area.)

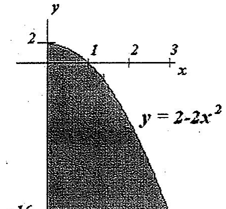
   (b) The Mean Value Theorem states that there exists at least one value $x = c$ between 0 and 2 such that $f(c)$equals to the average value of f. Find the value(s) of c.

9. [Fall 2008] Find the *total area* bounded by the curve $y = x^2 - 3x + 2$ and the x-axis in the interval $[-1, 2]$.

10. [Spring 2008] Find the total area of the two regions enclosed between the graph of the function $y = 1 - \frac{x^2}{4}$ on the interval $[-2, 3]$ and the x-axis.

11. [Summer 2007]

   (a) Find the shaded area of



(b) Find the area under the curve $f(x) = |x-1| + 2$ and the x-axis for $0 \leq x \leq 2$.

12. Compute the followings:

(a) $\frac{d}{dx}\int_5^{x^2} \frac{\ln(z)}{\sqrt{z^4+9}}dz$ (c)If $F(x) = \int_{x-1}^{x+1} f(t)\,dt$ and $f(t) = \int_1^{t^2} \frac{\sqrt{w^4-1}}{w}dw$, find $F''(2)$.

(b) Suppose $\int_1^x f(t)\,dt = x^2 - 2x + 1$.Find $f(x)$. (d)$\frac{d}{dx}\int_0^{\pi} \frac{\sin(t)-\cos(t)}{\sqrt{2+\cos(t)}}dt$

13.

(a) Find the linearization $L(x)$of $f(x) = \int_0^{\ln x} \cos^2(t)\,dt$ at $x_0 = 1$.

(b) [Fall 2008] Let $g(x) = 3 + \int_1^{x^2} (1 + ln(t))\,dt$.

i. Find the *linearization* of $g(x)$ at $x = -1$.

ii. Find $g''(-1)$.

(c) Find a function f and a number $a \neq 0$ such that

$$\frac{e^9}{a} + \int_a^x f(t)\,dt = \frac{e^{x^2}}{x}.$$

14. [Spring 2005] Let $f : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to \mathbb{R}$ be a continuous function, $a\in\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ be a given number and let $y(x)$, with $x\in\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, be the function defined as:

$$y(x) = \int_a^x f(t)\cos(t)\sec(x)\,dt.$$

(a) Compute $\frac{dy}{dx}$. Justify your steps and simplify your answer.

(b) Use part (a) to show that $y(x)$is a solution for the following differential equation:

$$\frac{dy}{dx} - \tan(x)\,y = f(x).$$

(c) Compute $y(a)$.

15. [Spring 2009] Consider the function: $h(x) = \cos(x) + \int_0^{\sqrt{x}} t^3 \sin\left(t^2\right) dt$ on the interval $[0, \pi]$.

(a) State the FTC, parts I and II. (b) Calculate the derivative $\frac{dh}{dx}$. (c) Find the critical points of h in the interval.

16. [Fall 2003] Consider the function

$$g(x) = \int_1^x f(t)\,dt$$

If $f(t)$ has a **positive derivative** for all values of t and $f(2) = 0$, decide which of the following statements are **always true**, which are **always false** and which you have not enough information to make a claim. There's no need to justify your claims. Simply write the statement letter along with your claim: TRUE, FALSE or NOT ENOUGH INFO.

(a) $g(x)$ is a differentiable function of x.

(b) $g(x)$ is a continuous function of x.

(c) The graph of $g(x)$ has a horizontal tangent at $x = 2$.

(d) $g(x)$has a local maximum at $x = 2$.

(e) The graph of $g(x)$ has an inflection point at $x = 2$.

17. [Summer 2007] Consider the integral $I = \int_{-2}^{1} \left(x^4 + 1\right) dx$. Suppose we want to estimate the value of I by using numerical integration.

(a) Estimate I using a **left hand** approximation with 3 rectangles (Riemann Sum)

(b) Estimate I by using 3 trapezoids (Trapezoidal Rule)

(c) Evaluate $I = \int_{-2}^{1} \left(x^4 + 1\right) dx$ exactly

(d) Find the minimum number of subintervals n needed to approximate the integral with an error of magnitude less than .0108 using the trapezoidal rule.

18. [Spring 2009] Consider the integral $\int_1^3 \frac{1}{x}dx$.

(a) Estimate the value of the integral using $n = 4$ rectangles and left-hand end-points (you do not need to simplify your answer).

(b) Estimate the same integral using the trapezoidal sum for $n = 4$ (you do not need to simplify your answer).

(c) Which of the above is an underestimate/overestimate? Justify your answer.(Hint: you may want to sketch the graph of the function, together with the respective rectangles and trapezoids, and explain.)

(d) How large do you have to make n to be sure that the corresponding trapezoidal estimate is within 3×10^{-4} of the real value of the integral?
Hint: The trapezoidal sum and error bound for the trapezoidal rule are $T_n = \frac{h}{2}(y_0 + 2y_1 + ... + 2y_{n-1} + y_n)$ and $|E_T| \leq \frac{b-a}{12}h^2 M$.

19. [Fall 2006]

(a) Find $f^{-1}(x)$ when $f(x) = x^2 - 2x + 1$, $x \geq 1$

(b) What are the domain and range of the $f^{-1}(x)$found in part (a)?

(c) Find y' when $y = \frac{\ln x}{1+\ln x}$. Simplify your answer.

20. [Fall 2008]

(a) Find $y(x)$, given $\ln(y^2 - 1) - \ln(y + 1) = \ln(sin(x))$.

(b) State the *Mean Value Theorem for Definite Integrals.* Be sure to include the hypothesis of the theorem in your answer.

(c) Suppose $f(x) = e^{-x} - 3x - \sin(x)$, is $f(x)$ invertible? Justify your answer.

21. [Spring 2009] Consider the function $f(x) = \frac{x}{x-1}$.

(a) What is the domain of f ?

(b) Is f one-to-one? If yes, calculate the inverse function $f^{-1}(x)$.

(c) What are the domain and range of f^{-1}?

22. [Fall 2007]

(a) Which of the functions $f(x) = x^3 - 1$ or $g(x) = x^4 + 2$ has an inverse? Justify your answer. Find a formula for inverse of this function.

(b) Graph this function and its inverse.

23. [Fall 2007]

(a) Find numbers x and y such that $e^x = 2$ and $2^y = e$.

(b) What is the relation between x and y?

(c) Based on (a), evaluate e^{5x}.

(d) Solve the following equation for z: $\ln(10 \cdot \ln z) = \ln(5t^2)$.

24. [Fall 2007]

(a) Using logarithmic differentiation, calculate the derivative of :

$$y = \sqrt{\frac{(x+1)^{10}}{(2x+1)^5}}$$

(b) Evaluate $\frac{dy}{dx}$ at $x = 0$.

25. [Fall 2007]

(a) Find the linearization of $\ln(1 + x)$, valid near $x = 0$.

(b) Based on your result in (a), find an approximate value for $\ln(1.1)$.

(c) Write $\ln(1 + x)$ in terms of a denite integral whose limit(s) depend on x.

(d) Use the trapezoidal rule with $N = 1$ to find a different approximate value for $\ln(1.1)$.

(e) Use the error estimate for the trapezoidal rule to find an upper bound on the error of your approximation in (d).

26. Calculate $\frac{dy}{dx}$ for each of the following:

(a) $(x + y)^x = \pi$

(b) $y = x^{\ln x}$

(c) $y = \int_{e^x}^{\pi} t \ln(t)\, dt$

(d) $y = x^x$

(e) $y = (1 - 2y)^x$

(f) $y = (\sin(x))^{\ln x}$