

$$1. a) \quad y = \sqrt{2 - \sqrt{3t}}$$

$$\frac{dy}{dt} = \frac{1}{2} (2 - \sqrt{3t})^{-1/2} \cdot (-\frac{1}{2})(3t)^{-1/2} \cdot 3$$

$$b) \quad f(\theta) = \left(\frac{\sin \theta}{1 + \cos \theta} \right)^2$$

$$\frac{df}{d\theta} = 2 \left(\frac{\sin \theta}{1 + \cos \theta} \right) \cdot \left(\frac{\cos \theta (1 + \cos \theta) + \sin \theta \cdot \sin \theta}{(1 + \cos \theta)^2} \right)$$

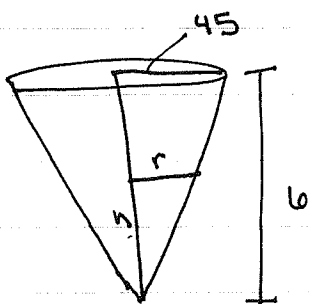
$$c) \quad \lim_{x \rightarrow 0} \frac{3}{3} \cdot \frac{\sin(3x)}{4x} = \frac{3}{4} \cdot \lim_{x \rightarrow 0} \frac{\sin 3x}{3x} = \boxed{\frac{3}{4}}$$

$$d) \quad 2xy + \pi \sin y = 2\pi$$

$$2 \cdot y + 2x \frac{dy}{dx} + \pi \cos y \cdot \frac{dy}{dx} = 0$$

$$\boxed{\frac{dy}{dx} = \frac{-2y}{2x + \pi \cos y}}$$

2.



$$\frac{dV}{dt} = -50 \text{ m}^3/\text{min}$$

$$\frac{dh}{dt} = ?$$

$$\frac{h}{6} = \frac{r}{45}$$

$$V = \frac{1}{3} \pi r^2 h$$

$$r = \frac{45h}{6}$$

$$V = \frac{1}{3} \pi \left(\frac{45h}{6} \right)^2 h$$

$$V = \frac{\pi \cdot \overset{675}{\cancel{45^2}}}{6} h^3$$

$$\frac{dV}{dt} = \frac{675\pi \cdot \cancel{2}}{2} h^2 \frac{dh}{dt}$$

$$\frac{dV}{dt} = \frac{675\pi}{2} \cdot h^2 \frac{dh}{dt}$$

$$-50 = \frac{675\pi}{2} \cdot 25 \cdot \frac{dh}{dt}$$

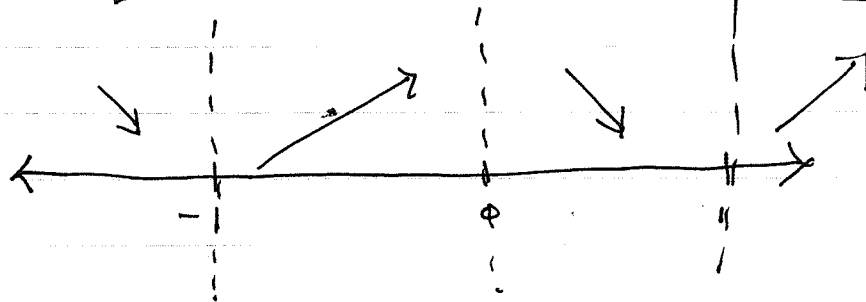
$$\frac{dh}{dt} = \frac{-50 \cdot 2}{675 \cdot \pi \cdot \cancel{25}} = \boxed{\frac{-4}{675\pi}}$$

3. $f(x) = x^2(x^2 - 2) = x^4 - 2x^2$

a) $f'(x) = 4x^3 - 4x = x(4x^2 - 4)$

$x=0, 1, -1$ critical points

b)



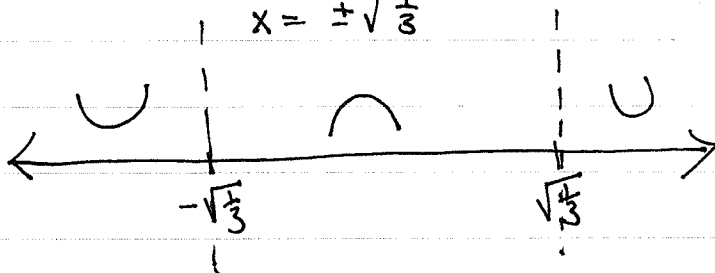
$f'(-2) = -2(16-4) = -$
 $f'(-\frac{1}{2}) = -\frac{1}{2}(1-4) = +$
 $f'(\frac{1}{2}) = \frac{1}{2}(1-4) = -$
 $f'(2) = 2(16-4) = +$

inc. on
 ~~$(-\infty, -1) \cup (0, 1)$~~
 $(-1, 0) \cup (1, \infty)$

c) Local Min $x = -1, 1$ 1st deriv. test
 Local Max $x = 0$ " " "

d) $f'' = 12x^2 - 4 = 0$

$12x^2 = 4$
 $x^2 = \frac{4}{12}$
 $x = \pm\sqrt{\frac{1}{3}}$



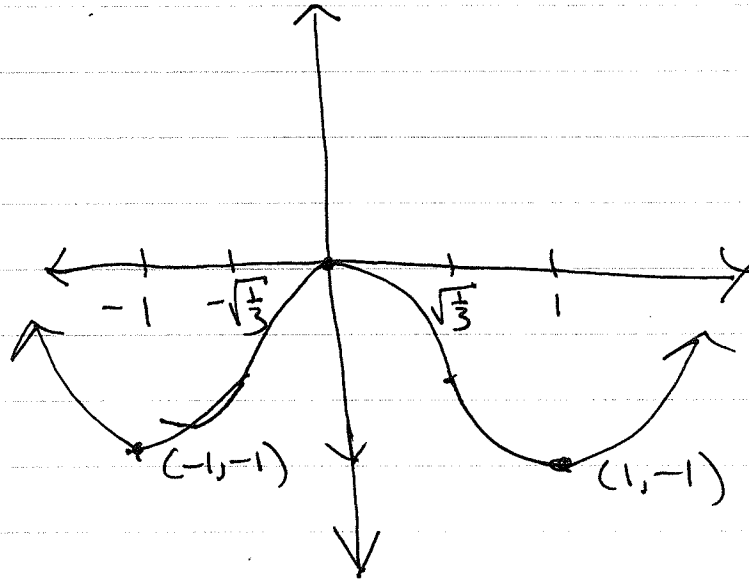
$f''(-1) = 12-4 = +$
 $f''(0) = -4 = -$
 $f''(1) = 12-4 = +$

c.u. $(-\infty, -\frac{1}{\sqrt{3}}) \cup (\frac{1}{\sqrt{3}}, \infty)$
 c.D. $(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$

$$f(x) = x^2(x^2 - 2) = 1(1 - 2) = -1$$

e) Infl. Pts $\pm\sqrt{\frac{1}{3}}$ b/c change in concavity

f)



4.

$$f(x) = \frac{4x^2}{x^2 - 4}$$

V.A.:	$x = \pm 2$
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$$\lim_{x \rightarrow -2^-} \frac{4(4)}{0} = \pm\infty$$

is $\boxed{-\infty}$

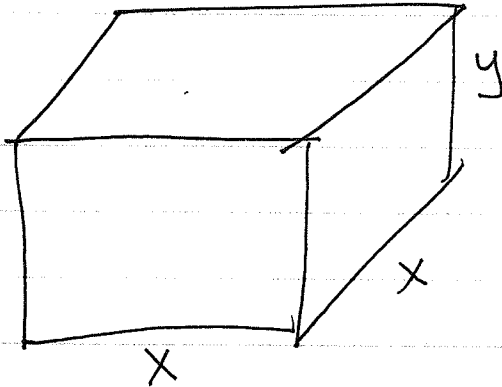
$$\lim_{x \rightarrow 2^+} = \frac{4(4)}{0} = \boxed{+\infty}$$

$$\text{H.A.}: \lim_{x \rightarrow \infty} \frac{4x^2}{x^2 - 4} = 4$$

$$\lim_{x \rightarrow -\infty} \frac{4x^2}{x^2 - 4} = 4$$

$y = 4$	H.A.
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5.



$$V = x^2 y = 32$$

$$y = \frac{32}{x^2}$$

$$S.A. = x^2 + 4xy$$

$$S = x^2 + 4 \cdot x \left(\frac{32}{x^2} \right)$$

$$S = x^2 + \frac{128}{x}$$

$$\frac{dS}{dx} = 2x - \frac{128}{x^2} = 0$$

$$2x = \frac{128}{x^2}$$

$$x^3 = 64$$

$$y = 2$$

$$x = 4$$