1. a) Find $\int \frac{t^2}{2} + 4t^3 \, dt$

$$\boxed{= \frac{t^3}{6} + t^4 + C}$$

b) Find $\int 1 + \tan^2 x \, dx$ trig id: $1 + \tan^2 x = \sec^2 x$

$= \int \sec^2 x \, dx$

$$\boxed{= \tan x + C}$$

c) Find $\int \frac{e^{-\sqrt{x}}}{\sqrt{x}} \, dx$

$u = -x^{1/2}$

$du = \frac{-1}{2\sqrt{x}} \, dx$

$-2\,du = \frac{dx}{\sqrt{x}}$

$= -2\int e^u \, du$

$= -2e^u + C$

$= \boxed{-2e^{-\sqrt{x}} + C}$

d) Find $\int_{-1}^{-1/2} t^{-2}\sin^2\left(1+\frac{1}{t}\right)dt$

$u = 1 + \frac{1}{t}$

$du = \frac{-1}{t^2}\,dt$

$-du = \frac{dt}{t^2}$

$t=-1 \Rightarrow u=0$

$t=-\frac{1}{2} \Rightarrow u=-1$

$$= -\int_0^{-1} \sin^2 u\,du$$

$$= \int_{-1}^{0} \sin^2 u\,du$$

$$= \int_{-1}^{0} \frac{1-\cos 2u}{2}\,du$$

$$= \frac{1}{2}u - \frac{\sin 2u}{4}\Big|_{-1}^{0}$$

$$= \left(\frac{1}{2}\cdot 0 - \frac{\sin(2\cdot 0)}{4}\right) - \left(-\frac{1}{2} - \frac{\sin(-2)}{4}\right)$$

$$= (0 - 0) + \frac{1}{2} + \frac{\sin(-2)}{4}$$

$$= \boxed{\frac{1}{2} + \frac{\sin(-2)}{4}}$$

e) Find $\frac{dy}{dr}$ when $y = e^{\sin r}(\ln r^3)$

$$y = e^{\sin r} \cdot 3\ln r$$

$$\frac{dy}{dr} = (e^{\sin r} \cdot \cos r) \cdot 3\ln r + e^{\sin r} \cdot \frac{3}{r}$$

$$\frac{dy}{dr} = e^{\sin r}\left[3\cos r + \frac{3}{r}\right]$$

2. $a(t) = -4\sin 2t$ $v(0)=2,$ $s(0) = -3$

a) $\int a(t)\,dt = \int -4\sin 2t \; dt$

$$v(t) = \frac{+4\cos 2t}{2} + c_1$$

$$v(t) = 2\cos 2t + c_1$$ I.C. $v(0)=2$

$$2 = 2\cos(0) + c_1$$

$$2 = 2 \cdot 1 + c_1$$

$$\therefore \quad c_1 = 0$$

$$v(t) = 2\cos 2t$$ velocity eqn.

$$\int v(t)\,dt = \int 2\cos 2t \; dt$$

$$s(t) = \frac{2\sin 2t}{2} + c_2$$

$$s(t) = \sin 2t + c_2$$ I.C. $s(0) = -3$

$$-3 = \sin(0) + c_2$$

$$\therefore \quad c_2 = -3$$

$$\boxed{s(t) = \sin 2t - 3}$$ position function

b) for what $t$ in $[0, \pi]$ is particle at rest?

$v(t) = 0$ is at rest.

$$v(t) = 2\cos 2t = 0$$
$$\cos 2t = 0$$

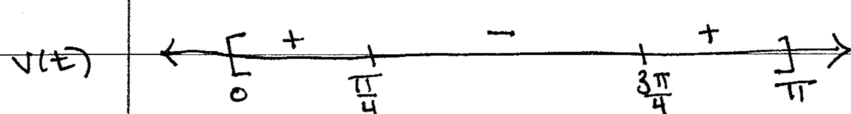
when $2t = \frac{\pi}{2}, \frac{3\pi}{2}$

$t = \frac{\pi}{4}, \frac{3\pi}{4}$

$v(t)$ is zero at $\boxed{t = \frac{\pi}{4}, \frac{3\pi}{4}}$

c) Particle is moving forward when $v(t) > 0$

backward when $v(t) < 0$




Forward $(0, \frac{\pi}{4}) \cup (\frac{3\pi}{4}, \pi)$

Backward $(\frac{\pi}{4}, \frac{3\pi}{4})$

3. a) FTC part 1

$f(x)$ continuous ~~on~~ on $[a, b]$ ;

$F(x) = \int_0^x f(t)\,dt$ has derivative on $[a,b]$

and $\frac{dF}{dx} = \frac{d}{dx}\int_0^x f(t)\,dt = f(x)$.

FTC Part 2

$f(x)$ continuous on $[a, b]$
$F(x)$ any antiderivative of $f(x)$.

Then $\int_a^b f(x)\,dx = F(b) - F(a)$

3. b) Find Linearization of

$$f(x) = 2 - \int_2^{x+1} \frac{9}{1+t}\, dt \qquad \text{at} \qquad x=1$$

$$f(1) = 2 - \int_2^{2} \frac{9}{1+t}\, dt = 2$$

$$f'(x) = -\frac{9}{1+(x+1)} = \frac{-9}{x+2}$$

$$f'(1) = \frac{-9}{1+2} = -3$$

$$\therefore \quad \boxed{L(x) = 2 - 3(x-1)}$$

c) express $\lim\limits_{\|P\| \to 0} \sum\limits_{k=1}^{n} \frac{1}{1-c_k} \Delta x_k$ where P is a partition of $[2,3]$ as an integral

$$\boxed{= \int_2^3 \frac{1}{1-x}\, dx}$$

4. a) Given $\int_0^1 \frac{4}{1+x^2}\,dx = \pi$, estimate $\pi$ numerically

with Trapezoid rule & $n=2$ subintervals.

$$h = \frac{1-0}{2} = \frac{1}{2}$$

$$T = \frac{\frac{1}{2}}{2}\left(y_0 + 2y_1 + y_2\right)$$

$x_0 = 0$ $\qquad$ $y_0 = 4$

$x_1 = \frac{1}{2}$ $\qquad$ $y_1 = \frac{4}{1+\frac{1}{4}} = \frac{4}{\frac{5}{4}} = \frac{16}{5}$

$x_2 = 1$ $\qquad$ $y_2 = \frac{4}{2} = 2$

$$T = \frac{1}{4}\left(4 + 2\left(\frac{16}{5}\right) + 2\right)$$

$$= 1 + \frac{8}{5} + \frac{1}{2}$$

$$= \frac{10}{10} + \frac{16}{10} + \frac{5}{10}$$

$$\boxed{= \frac{31}{10} \approx 3.1}$$

b) approximation error is $E_T = \pi - 3.1$

also acceptable: $|E|_T < \frac{1}{6}$