

5. (24 points) Let

$$F(x) = \int_0^{\sin x} \frac{dt}{\sqrt{1-t^2}} \quad |x| < \frac{\pi}{2}$$

(a) Find $\frac{dF}{dx}$.

(b) Evaluate $F\left(\frac{\pi}{4}\right)$.

6. (20 points) You are designing a 1000-cm³ right circular cylindrical containment tank and you need to minimize the cost of material. The cost of material for the side-walls of the tank is \$1 per square centimeter and the cost of the circular top and bottom is \$2 per square centimeter. What are the dimensions of the tank that will minimize the cost of material?

7. (16 points) A cup of hot soup is cooled from 90°C to 60°C after 10 minutes in a room whose temperature was 20°C. Use Newton's law of cooling

$$T - T_S = (T_0 - T_S)e^{-kt},$$

to find how much longer it will take the soup to cool to 35°C. (Hint: $\frac{\ln(\frac{4}{7})}{-10} \approx ??$.)