APPM 1350 **Midterm #1** **Fall 2010**

On the front of your bluebook, please write: a grading key, your name, student ID, section, and instructor's name (Biesterfeld, Curry, Curtis, Dougherty, Nelson) . This exam is worth 100 points and has 6 questions. **Show all work!** Answers with no justification will receive no points. Please begin each problem on a new page. No notes, calculators, or electronic devices are permitted.

1. (25 points) Evaluate each of the following and show all supporting work. If a limit does not exist, clearly state that fact and explain your reasoning.

   $(a)\quad \lim_{x\to 4} \sqrt{19+\sqrt{9x}}$ $\qquad (b)\quad \lim_{x\to 0} \dfrac{\sin(3x)(1-\cos^2 x)}{x^3}$ $\qquad (c)\quad \lim_{x\to -6} \dfrac{\frac{1}{6}+\frac{1}{x}}{x+6}$

   $(d)\quad \lim_{u\to 2^+} \dfrac{2+u}{u^2-4}$ $\qquad (e)\quad \lim_{v\to 4^+} \dfrac{4-v}{|4-v|}$

2. (8 points) True or False. If the statement is true, write out the word TRUE. If the statement is false, write the word FALSE. For this problem only, no explanation is needed.

   (a) $\dfrac{d}{dx}|x^3+x| = |3x^2+1|$
   (b) If $f(x)$ is an odd function and $g(x)$ is an even function, then $f(x)g(x)$ is even.

3. (16 points) The equation of motion of a particle is $s(t) = t^3 - 9t^2 + 15t$ where $s(t)$ is in meters and t is in seconds.

   (a) Find the velocity and acceleration.
   (b) Find the acceleration when the velocity is zero.
   (c) When is the particle moving in the positive direction?
   (d) Find the total distance traveled during the first 10 seconds.

4. (15 points) Let $f(x) = \begin{cases} x\sin(1/x) & \text{if } x < 0, \\ x^2 & \text{if } 0 \le x < 1, \\ \sin(2x) & \text{if } x \ge 1. \end{cases}$

   (a) Give the definition for a function $f(x)$ to be continuous at a point c.
   (b) Is $f(x)$ continuous or discontinuous at $x = 0$? At $x = 1$? Explain.
   (c) Using interval or set notation, write down the set of x where $f(x)$ is continuous.

5. (18 points)

   (a) Give the definition of the derivative of a function $y = g(x)$.
   (b) Use the definition of the derivative to find $g'(x)$ for $g(x) = 3x + \dfrac{2}{x}$.
   (c) Find the equation of the tangent line to $g(x) = 3x + \dfrac{2}{x}$ at $x = -1$.

TURN PAGE OVER FOR PROBLEM #6

6. (18 points) Consider a function $y = f(x)$ for which all of the following is known:

- $f(0) = 1$
- $\lim_{x \to 0^-} f(x) = 1$ and $\lim_{x \to 0^+} f(x) = 1$
- $\lim_{h \to 0^-} \dfrac{f(h) - f(0)}{h} = -1$ and $\lim_{h \to 0^+} \dfrac{f(h) - f(0)}{h} = 1$
- $\lim_{x \to \infty} f(x) = 2$ and $\lim_{x \to -\infty} f(x) = 2$

Using the above information, answer the following questions. Explain your reasoning for each part.

(a) Is $x = 0$ in the domain of f?

(b) Does $\lim_{x \to 0} f(x)$ exist?

(c) Is $f(x)$ continuous at $x = 0$?

(d) Is $f(x)$ differentiable at $x = 0$?

(e) Are there any horizontal asymptotes?

(f) Sketch a graph of a function $y = f(x)$ which satisfies the above requirements and is an even function.