1\. a. $\lim_{x \to 4} \sqrt{19+\sqrt{9x}} = \sqrt{19+\sqrt{9 \cdot 4}} = \sqrt{19+6} = \sqrt{25} = 5.$

b. $\lim_{x \to 0} \dfrac{\sin(3x)(1-\cos^2 x)}{x^3} = \lim_{x \to 0} \dfrac{\sin(3x) \cdot \sin^2 x}{x^3}$

$= \lim_{x \to 0} \dfrac{\sin(3x)}{x} \cdot \dfrac{\sin x}{x} \cdot \dfrac{\sin x}{x}$

$= \lim_{x \to 0} 3 \cdot \dfrac{\sin(3x)}{3x} \cdot \lim_{x \to 0} \dfrac{\sin x}{x} \cdot \lim_{x \to 0} \dfrac{\sin x}{x} = 3 \cdot 1 \cdot 1 \cdot 1 = 3$

c. $\lim_{x \to -6} \dfrac{\frac{1}{6} + \frac{1}{x}}{x+6} = \lim_{x \to -6} \dfrac{1}{x+6} \cdot \left[\dfrac{1}{6} + \dfrac{1}{x}\right] = \lim_{x \to -6} \dfrac{1}{\cancel{x+6}} \cdot \dfrac{\cancel{x+6}}{6x} = -\dfrac{1}{36}$

d. $\lim_{u \to 2^+} \dfrac{u+2}{u^2-4} = \lim_{u \to 2^+} \dfrac{u+2}{(u+2)(u-2)} = \lim_{u \to 2^+} \dfrac{1}{u-2} = \infty.$

e. $|4-v| = \begin{cases} 4-v, & \text{if } 4-v \geq 0 \\ -(4-v), & \text{if } 4-v < 0 \end{cases} = \begin{cases} 4-v, & \text{if } 4 \geq v \\ v-4, & \text{if } 4 < v \end{cases}$

$\lim_{v \to 4^+} \dfrac{4-v}{|4-v|} = \lim_{v \to 4^+} \dfrac{4-v}{v-4} = \lim_{v \to 4^+} -\dfrac{\cancel{(v-4)}}{\cancel{v-4}} = -1.$

2 a. False, $|x^3+x|$ is not ~~even~~ differentiable at $x=0$.
There is a cusp.

b. False

f odd $\Rightarrow f(-x) = -f(x)$

g even $\Rightarrow g(-x) = g(x)$

$f(-x)\,g(-x) = -f(x)\cdot g(x) = -\left[f(x)\cdot g(x)\right]$

so $f(x)\cdot g(x)$ is an odd function.

3. $s(t) = t^3 - 9t^2 + 15t$

a. $v(t) = s'(t) = 3t^2 - 18t + 15$

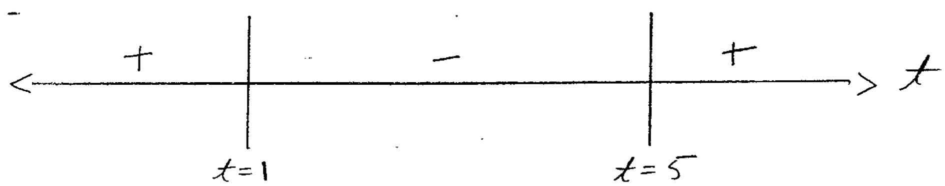
$a(t) = v'(t) = s''(t) = 6t - 18$

b. $v(t) = 0 = 3t^2 - 18t + 15 = 3(t^2 - 6t + 5) = 3(t-5)(t-1)$

$\Rightarrow$ v is zero at $t = 1, 5$.

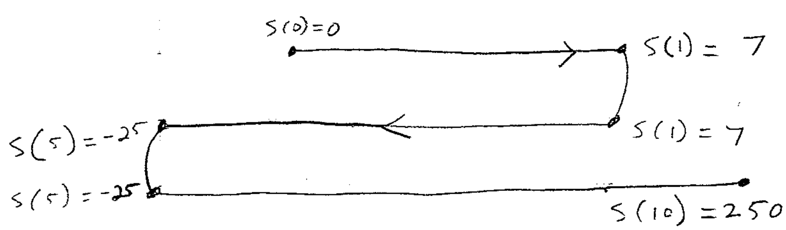
$a(1) = -12, \quad a(5) = 12$

c. The particle is moving in the positive direction when $v(t) > 0$. We already know that $v(t) = 0$ at $t = 1, 5$.



test $t = 0 \Rightarrow v(0) = 15$ so $v(t) > 0$ when $t \in (-\infty, 1) \cup (5, \infty)$

d. Note that the direction changes everytime $v = 0$.



distance traveled $= 7$

distance traveled $= 7 - (-25) = 32$

distance traveled $= 250 - (-25) = 275$

Total Distance $= 7 + 32 + 275 = 314$ meters

4. $$f(x) = \begin{cases} x \sin(1/x), & \text{if } x < 0 \\ x^2, & \text{if } 0 \le x < 1 \\ \sin(2x), & \text{if } x \ge 1 \end{cases}$$

a. A function $f(x)$ is said to be continuous at a point

c if $\lim_{x \to c} f(x) = f(c)$. Note that the limit must exist for this to make sense.

b. at $x = 0$,

$$-1 \le \sin(1/x) \le 1$$

$\lim_{x \to 0^+} x \sin(1/x) = 0$ because $\underset{\downarrow \atop 0}{-x} \le \underset{\downarrow \atop 0}{x \sin(1/x)} \le \underset{\downarrow \atop 0}{x}$

by the sandwich theorem, as $x \to 0^+$.

$\lim_{x \to 0^-} x \sin(1/x) = 0$ because $\underset{\downarrow \atop 0}{x} < \underset{\downarrow \atop 0}{x \sin(1/x)} \le \underset{\downarrow \atop 0}{-x}$

by the sandwich theorem again, as $x \to 0^-$.

In addition $f(0) = 0$. Since

$$\lim_{x \to 0^+} f(x) = 0 = \lim_{x \to 0^-} f(x) = 0 = f(0) = 0$$

we have that $f(x)$ is continuous at $x = 0$.

For $x = 1$, (remember in Calculus, we always work in radians)

$f(x)$ is discontinuous because

$$1 = \lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x^2 \neq \lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \sin(2)$$

because the sine function is one at $\pi/2$ and then $5\pi/2$ and clearly $x = 2$ is strictly in between them.

c. $f(x)$ is cont. on $(-\infty, 1) \cup (1, \infty)$
or $\{x \in \mathbb{R} \mid x \neq 1\}$.
(all real numbers except one)

5. a. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ or $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

b. $g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$

$= \lim_{h \to 0} \frac{\left[3(x+h) + \frac{2}{x+h}\right] - \left[3x + \frac{2}{x}\right]}{h}$

$= \lim_{h \to 0} \frac{3(x+h) - 3x}{h} + \frac{\frac{2}{x+h} - \frac{2}{x}}{h}$

$= \lim_{h \to 0} \frac{3x + 3h - 3x}{h} + \frac{2x - 2x - 2h}{h\,x\,(x+h)}$

$= \lim_{h \to 0} \frac{3\cancel{h}}{\cancel{h}} + \frac{-2\cancel{h}}{\cancel{h}\,x\,(x+h)}$

$= 3 - \frac{2}{x^2}$.

c. at $x = -1$, $g(-1) = -3 - 2 = -5 \Rightarrow (-1, -5) = P$
at $x = -1$, $g'(-1) = 3 - 2 = 1 \Rightarrow 1 = m$

Using point-slope $\Rightarrow$ $y + 5 = x + 1$
or $y = x - 4$.

6. a) Yes, because $f(0)=1$ is defined.

b) Yes, because $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x) = 1$

c) Yes, because $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x) = f(0) = 1$

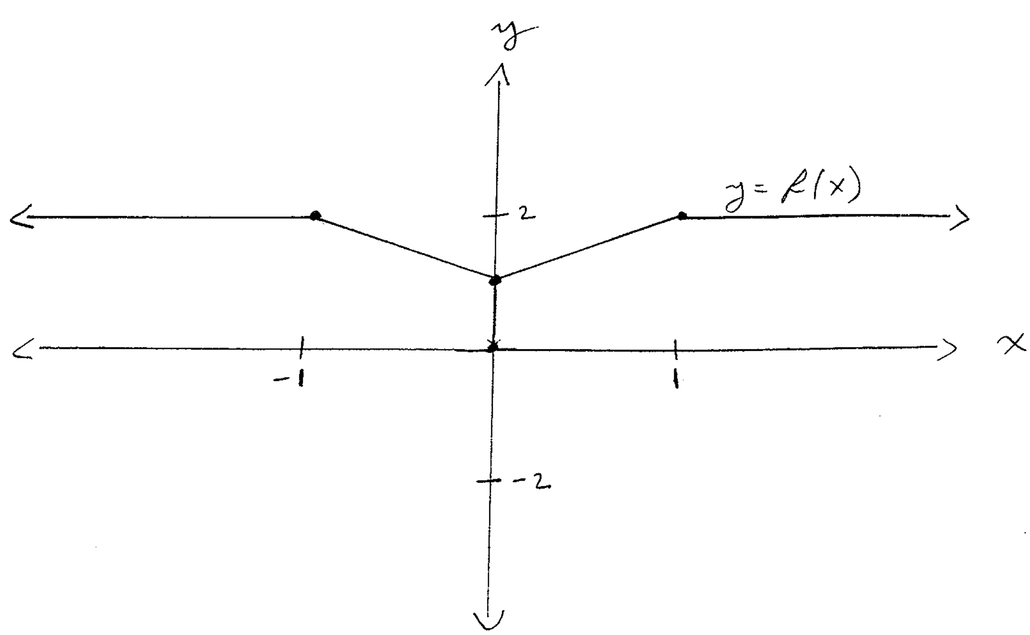
d) No, because

$$-1 = \lim_{h \to 0^-} \frac{f(h)-f(0)}{h} \neq \lim_{h \to 0^+} \frac{f(h)-f(0)}{h} = 1$$

e) Yes, $y=2$ is a horizontal asymptote

because $\lim_{x \to \pm\infty} f(x) = 2$.

f)



Multiple solutions are possible for this graph.