

**CAREFULLY PRINT, on the front of your bluebook: a grading key, your name, student ID, section, and instructor's name (Biesterfeld, Curry, Curtis, Dougherty, Nelson) .** This exam is worth 100 points and has 5 questions. **Show all work!** Answers with no justification will receive no points. Please begin each problem on a new page. No notes, calculators, or electronic devices are permitted.

1. (21 points) Find  $dy/dx$  for the following functions. **You do not need to simplify any of your answers for this problem.**

(a)  $y = (1 - x^4)^3(1 + x^2)^5$

(b)  $y = \frac{1 + xf(x)}{1 + \sqrt{x}}$  (Assume  $f(x)$  is differentiable.)

(c)  $y = \sin^3(\cos(x^3 + 4x))$

2. (24 points) Consider the function  $f(x) = \frac{8x^3}{(x - 1)^3}$ .

(a) Find all asymptotes for  $f(x)$ . Justify your answer using the appropriate limits.

(b) The first derivative of  $f(x)$  is  $f'(x) = \frac{-24x^2}{(x - 1)^4}$ . Using the  $f'(x)$  provided, for what intervals on the  $x$ -axis is  $f(x)$  increasing? Decreasing? Also report the coordinates of the local maximums and minimums for  $f(x)$ , if any.

(c) The second derivative of  $f(x)$  is  $f''(x) = \frac{48x(x + 1)}{(x - 1)^5}$ . For what intervals on the  $x$ -axis is  $f(x)$  concave up? Concave down? Also report the coordinates of the inflection points for  $f(x)$ , if any.

(d) Using the information in (a)-(c), carefully sketch  $f(x)$ . Be sure to clearly label any asymptotes and the coordinates for any maximums, minimums, and inflection points.

3. (18 points) Consider the equation  $x^2y + 2x = 2 - \tan y$ .

- (a) Find  $dy/dx$  and  $d^2y/dx^2$  at the point  $(1, 0)$ .
- (b) Find the linearization of the equation at the point  $(1, 0)$ . Use the linearization to estimate the  $y$  value when  $x = 1.1$ .
- (c) Does the approximation in part (b) overestimate or underestimate the value of  $y$ ? Explain.
4. (16 points) A street light is mounted at the top of a 15-ft-tall pole. A man, 6 ft tall, walks away from the pole with a speed of 5 ft/s along a straight path. How fast is the tip of his shadow moving when he is 40 ft from the pole?
5. (21 points) Short answer questions. Please be sure to state any theorem you use.
- (a) At time  $t = 0$  a particle's velocity is 30 meters per minute. The particle's velocity increases smoothly so that, ten seconds later, its velocity is 50 meters per minute. Explain why the **acceleration** was 120 meters per minute squared at some time between the two times.
- (b) Does the function  $g(x) = \frac{-2x^2 + 5x - 1}{2x - 1}$  have a slant asymptote? If so, find it. If not, explain why a slant asymptote does not exist.
- (c) How many real roots does the function  $h(x) = 4x^5 + x^3 + 2x + 1$  have? Explain.