1. $y = (1-x^4)^3 (1+x^2)^5$

$$y' = \left[3(1-x^4)^2(-4x^3)\right]\left[(1+x^2)^5\right] + \left[(1-x^4)^3\right]\cdot\left[5(1+x^2)^4(2x)\right]$$

b. $y = \dfrac{1+x f(x)}{1+\sqrt{x}}$

$$y' = \frac{\left[f(x) + x f'(x)\right]\left[1+\sqrt{x}\right] - \left[1 + x f(x)\right]\left[\frac{1}{2\sqrt{x}}\right]}{(1+\sqrt{x})^2}$$

c. $y = \sin^3(\cos(x^3+4x))$

$$y' = 3\left(\sin(\cos(x^3+4x))\right)^2 \cdot \left(\cos(\cos(x^3+4x))\right)$$

$$\cdot\left(-\sin(x^3+4x)\right)\cdot(3x^2+4)$$

2. $f(x) = \dfrac{8x^3}{(x-1)^3}$

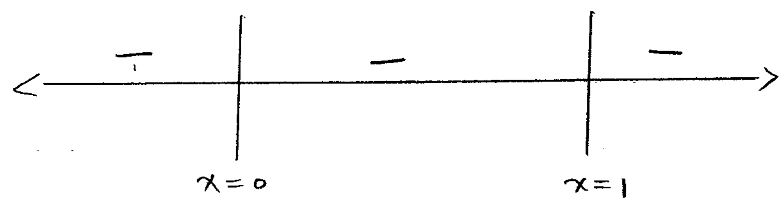
For H.A., $\lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \dfrac{8x^3}{(x-1)^3} = 8 = y$

so $y = 8$ is a H.A. There are no oblique/slant asymptotes.

For V.A., note that $\lim_{x \to 1} f(x) = \text{DNE}$

so $x = 1$ is the one and only V.A.

b. $f'(x) = \dfrac{-24x^2}{(x-1)^4}$



$f'(x) = 0 \Rightarrow x = 0$

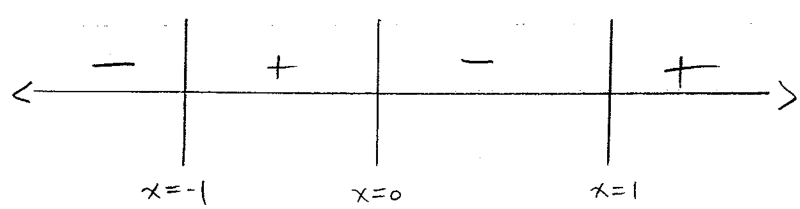
$f'(x) = \text{Undefined} \Rightarrow x = 1$ so $x = 0, 1$ are the only critical points. otherwise $f'(x) < 0$ always.

So $f(x)$ is decreasing on $(-\infty, 0) \cup (0,1) \cup (1, \infty)$.

c. $f''(x) = \dfrac{48x(x+1)}{(x-1)^5}$

$f''(x) = 0 \Rightarrow x = 0, -1$

$f''(x) = \text{Undef.} \Rightarrow x = 1$



$f''(-2) = 48(-2)(-1)/(-3)^5 > 0$, $f''(1/2) = 48(1/2)(3/2)/(-1/2)^5 < 0$

$f''(-1/2) = 48(-1/2)(1/2)/(-3/2)^5 > 0$, $f''(2) = 48(2)(3)/1 > 0$

(Remember, technically speaking $x=1$ is not an inflection because $x=1$ is not even in the domain of the function.)

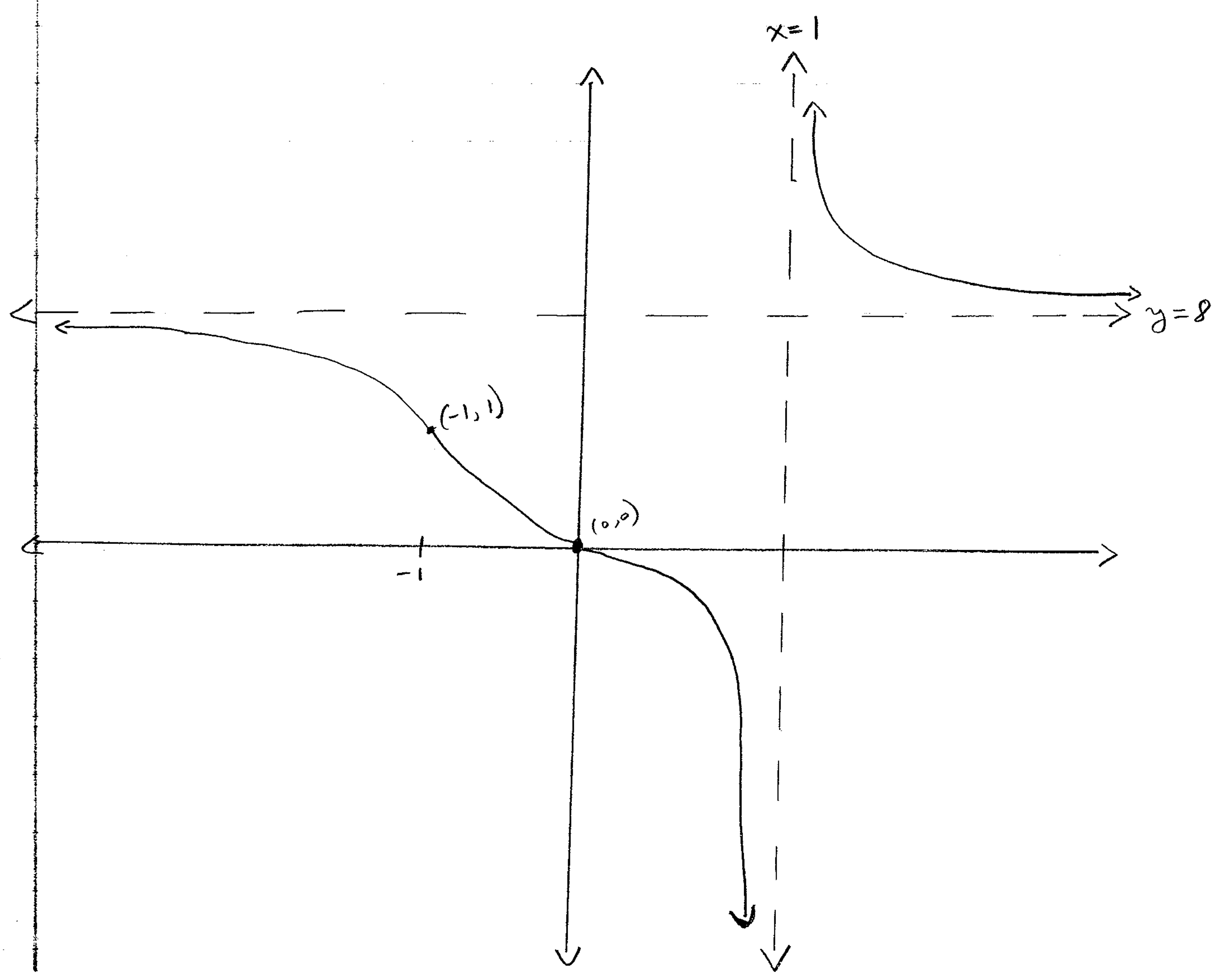
So $x=-1,0,1$ are the inflection points.

$f(x)$ is concave up on $(-1,0) \cup (1,\infty)$

and concave down on $(-\infty,-1) \cup (0,1)$.

We can also check the critical points from part b) using $f''(x)$.

$$\left.\begin{array}{l} f''(0)=0 \\ f''(1)=\text{Undef.} \end{array}\right\} \Rightarrow \text{No local max or mins.}$$



3. $x^2 y + 2x = 2 - \tan y$

$$2xy + x^2 y' + 2 = -(\sec^2 y)(y')$$

$$x^2 y' + (\sec^2 y)\, y' = -2 - 2xy$$

$$(x^2 + \sec^2 y)\, y' = -2 - 2xy$$

$$y' = \frac{-2-2xy}{x^2 + \sec^2 y} \Rightarrow y'\Big]_{(1,0)} = \frac{-2}{2} = -1$$

$$y'' = \frac{\left[\left(-2\left(xy' + y\right)\right)\right]\left[x^2 + \sec^2 y\right] - \left[2x + 2\sec^2 y \tan y \cdot y'\right]\cdot\left[-2 - 2xy\right]}{\left[x^2 + \sec^2 y\right]^2}$$

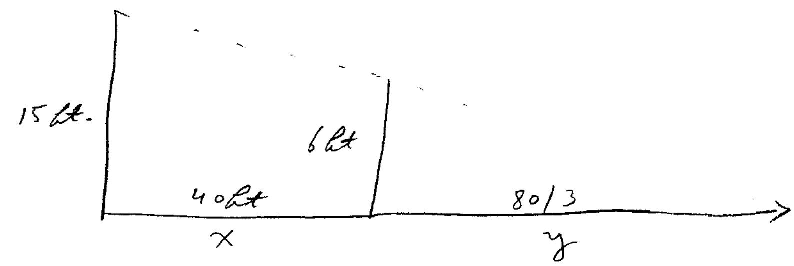
$$y''\Big|_{(1,0)} = \frac{\left[-2(1)(-1) + 0\right]\left[1+1\right] - \left[2\right]\left[-2\right]}{(1+1)^2} = \frac{2(2)+4}{2^2} = 2$$

b. $m = -1$, @ $(1,0)$ $\Rightarrow$ $y = -(x-1) = 1 - x$

$y(1.1) = -0.1$

c. Since $y'' > 0$, the graph is concave up so the tangent line is below the graph. Hence the linearization gives us an underestimate.

4.



$$\frac{15}{6} = \frac{x+y}{y} \Rightarrow 15y = 6x + 6y \Rightarrow 9y = 6x$$

$$\Rightarrow y = 2/3\, x$$

$$\frac{d}{dt}(x+y) = \frac{d}{dt}\left(x + \frac{2}{3}x\right) = \frac{d}{dt}(5/3\, x) = \frac{5}{3}\frac{dx}{dt}$$

$$= \frac{5}{3} \cdot 5 = 25/3 \text{ ft/s.}$$

5. $a = \frac{\Delta V}{\Delta t} = \frac{50 \text{ m/min} - 30 \text{ m/min}}{10 \text{ s} - 0 \text{ s}} = \frac{20 \text{ m/min}}{10 \text{ s}}$

$= \frac{20 \text{ m}}{\text{min}} \cdot \frac{1}{10 \cancel{\text{s}}} \cdot \frac{60 \cancel{\text{s}}}{1 \text{ min.}} = \frac{120 \text{ m}}{\text{min}^2}$

Since the acceleration was smooth, the velocity is a cont. and diff. function.

$$\frac{V(10) - V(0)}{10 - 0} = V'(c) = a(c) = 120$$

so using the MVT, we know that there must be $c \in (0,10)$ s.t. $a(c) = 120$.

b. Yes, $g(x)$ does have a slant asymptote at $y = 2 - x$

$$\begin{array}{r} -x + 2 \\ 2x - 1 \overline{\smash{)}\, -2x^2 + 5x - 1} \\ -(-2x^2 + x) \\ \hline 4x - 1 \\ -(4x - 2) \\ \hline 1 \end{array}$$

$\lim_{x \to \infty} f(x) - (2 - x) = \lim_{x \to \infty} \frac{-2x^2 + 5x - 1}{2x - 1} - (2 - x)$

$= \lim_{x \to \infty} 2 - x + \frac{1}{2x - 1} - 2 + x$

$= \lim_{x \to \infty} \frac{1}{2x - 1} = 0$ ✓

c. This function has only one real root because $f(-1) < 0$ and $f(0) > 0$ and $f'(x) = 20x^4 + 3x^2 + 2 > 0$ for all real x so it is always increasing. Therefore using the IVT there is at least one root but $f(x)$ is always increasing so it can cross the x-axis only once.