

CAREFULLY PRINT, on the front of your bluebook: a grading key, your name, student ID, section, and instructor's name (Biesterfeld, Curry, Curtis, Dougherty, Nelson) . This exam is worth 100 points and has 6 questions. **Show all work!** Answers with no justification will receive no points. Please begin each problem on a new page. No notes, calculators, or electronic devices are permitted.

1. (28 points) Evaluate the following expressions:

(a) $\int \frac{7x^2}{\sqrt{x^3 + 2}} dx$

(b) $\int_0^{\pi/2} \cos x \sin(\sin x) dx$

(c) $\int \frac{x^{4.1} + x - x^3}{x^4} dx$

(d) $\lim_{n \rightarrow \infty} \frac{1}{n} \left[\left(\frac{1}{n}\right)^9 + \left(\frac{2}{n}\right)^9 + \cdots + \left(\frac{n}{n}\right)^9 \right]$ (Hint: Think definite integral.)

2. (14 points) Evaluate each of the following derivatives and show your work. Simplify if possible.

(a) $\frac{d}{dx} \left(\int_4^{\sqrt{x}} t \sin(t^{10}) dt \right)$

(b) y' when $y = \ln \left(\frac{x+1}{\sqrt{x-2}} \right)$

3. (15 points) We wish to approximate $\int_0^1 x \sin x dx$.

(a) Write down a Riemann sum approximation to this integral by dividing $[0, 1]$ into four equal subintervals and using the left endpoint of each subinterval for evaluation. Just write the Riemann sum, DO NOT SIMPLIFY.

(b) Will your Riemann sum in part (a) overestimate or underestimate the definite integral? Explain briefly.

4. (10 points) Find the average value of $g(x) = \tan x$ on the interval $[0, \pi/4]$.

5. (18 points) Consider the function $h(x) = \sqrt{10 - 3x}$.

(a) What is the domain and range of $h(x)$?

(b) Show that $h(x)$ is one-to-one (and thus invertible).

(c) Find the inverse, $h^{-1}(x)$.

(d) State the domain and range of $h^{-1}(x)$.

6. (15 points) True/False. For this problem only, just write TRUE or FALSE, no explanation is required.

(a) $\int_a^b x f(x) dx = x \int_a^b f(x) dx$

(b) $\int_{-2}^1 \frac{1}{x^4} dx = -3/8$

(c) $\int f'(x)g'(x) dx = f(x)g(x) + C$

(d) If $f(x) \leq g(x)$ for all $a \leq x \leq b$, then $\int_a^b f(x) dx \leq \int_a^b g(x) dx$.

(e) If $f(x) \leq g(x)$ for all $a \leq x \leq b$, then $f'(x) \leq g'(x)$ for all $a \leq x \leq b$.