1. a. $\displaystyle\int \frac{7x^2\,dx}{\sqrt{x^3+2}}$ Let $u = x^3+2$

$du = 3x^2\,dx$

$$= \frac{7}{3}\int u^{-1/2}\,du = \frac{14}{3}u^{1/2} + C = \frac{14}{3}\sqrt{x^3+2} + C$$

b. $\displaystyle\int_0^{\pi/2} \cos x \sin(\sin x)\,dx$ Let $u = \sin x$

$du = \cos x\,dx$

$$= \int_0^1 \sin u\,du = -\cos(u)\Big]_0^1 = -\left[\cos(1) - 1\right] = 1 - \cos(1)$$

c. $$\int \frac{x^{4.1} + x - x^3}{x^4}\,dx = \int x^{0.1} + x^{-3} - x^{-1}\,dx$$

$$= \frac{x^{1.1}}{1.1} - \frac{x^{-2}}{2} - \ln|x| + C$$

d. $$\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^9 + \cdots + \left(\frac{n}{n}\right)^9\right] = \int_0^1 x^9\,dx = \frac{x^{10}}{10}\Big]_0^1 = \frac{1}{10}$$

2.a. $\frac{d}{dx}\int_4^{\sqrt{x}} t\sin(t^{10})\,dt = \sqrt{x}\sin\left((\sqrt{x})^{10}\right)\cdot\frac{d}{dx}\sqrt{x}$

$= \frac{\sqrt{x}\sin(x^5)}{2\sqrt{x}} = \frac{\sin(x^5)}{2}.$

b. $y = \ln\left(\frac{x+1}{\sqrt{x-2}}\right) \Rightarrow y' = \frac{\sqrt{x-2}}{x+1}\cdot\frac{(1)\sqrt{x-2} - (x+1)\frac{1}{2}(x-2)^{-1/2}}{x-2}$

$= \frac{\sqrt{x-2}}{x+1}\cdot\left[\frac{\sqrt{x-2}}{x-2} - \frac{x+1}{2(x-2)\sqrt{x-2}}\right]$

$= \frac{1}{x+1} - \frac{1}{2(x-2)}$

or $y = \ln(x+1) - \frac{1}{2}\ln(x-2)$

$y' = \frac{1}{x+1} - \frac{1}{2(x-2)}$

3. $x_i = 0, 1/4, 1/2, 3/4$ from $\frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4} = \Delta x$

$$S = \frac{1}{4} \cdot f(0) + \frac{1}{4} f(1/4) + \frac{1}{4} f(1/2) + \frac{1}{4} f(3/4)$$

$$= \frac{1}{4}\left[\frac{1}{4} \sin(1/4) + \frac{1}{2} \sin(1/2) + \frac{3}{4} \sin(3/4)\right]$$

b. $f'(x) = \sin x + x \cos x > 0$ on $(0, 1]$

$\Rightarrow$ f is increasing on this interval

$\Rightarrow$ using the left endpoints gives us an underestimate.

4. $f_{avg} = \frac{1}{b-a} \int_a^b f(x)\,dx = \frac{4}{\pi} \int_0^{\frac{\pi}{4}} \tan x\,dx$

$= \frac{4}{\pi} \Big[-\ln|\cos x| \Big]_0^{\pi/4} = \frac{-4}{\pi} \Big[\ln|\sqrt{2}/2| - \ln|1| \Big]$

$= -\frac{4}{\pi} \Big[-\frac{\ln(2)}{2} \Big] = \frac{\ln(4)}{\pi}$

5a. $h(x) = \sqrt{10-3x} \Rightarrow 10 - 3x \geq 0 \Rightarrow 10 \geq 3x \Rightarrow x \leq 10/3$

Domain of h is $(-\infty, 10/3]$, Range of h is $[0, \infty)$.

b. Pick any two x, y in the domain of h s.t. $x \neq y$

$\Rightarrow 3x \neq 3y \Rightarrow -3x \neq -3y \Rightarrow 10-3x \neq 10-3y$

$\Rightarrow \sqrt{10-3x} \neq \sqrt{10-3y} \Rightarrow f(x) \neq f(y)$

$\Rightarrow h$ is 1-1 $\Rightarrow h$ is invertible.

c. $y = \sqrt{10-3x} \Rightarrow y^2 = 10 - 3x \Rightarrow 3x = 10 - y^2$

$\Rightarrow x = \dfrac{10-y^2}{3} \Rightarrow h^{-1}(x) = \dfrac{10-x^2}{3}$

d. The domain and the range of h flip

Range of $h^{-1}(x)$ is $(-\infty, 10/3]$

Domain of $h^{-1}(x)$ is $[0, \infty)$.

6. a. False, because only constants can be pulled out of an integral and x is not a constant with respect to x.

b. False, because $f(x) = x^{-4}$ has a vertical asymptote at $x=0$, the evaluation thm doesn't apply. This integral cannot be evaluated.

c. False, because $\big(f(x)\,g(x)\big)' \neq f'(x)\,g'(x)$.

d. True, if $f \leq g \Rightarrow f-g \leq 0 \Rightarrow \int_a^b f-g \leq 0$

$\Rightarrow \int_a^b f \leq \int_a^b g$ on $[a, b]$.

e. False, let $f(x) = x$ and $g(x) = 10$ on $[0,1]$.

$f(x) = x \leq 10 = g(x)$ on $[0,1]$

but $f'(x) = 1 \nleq 0 = g'(x)$ on $[0,1]$.