

CAREFULLY PRINT, on the front of your bluebook: a grading key, your name, student ID, section, and instructor's name (Biesterfeld, Curry, Curtis, Dougherty, Nelson). This exam is worth 150 points and has 8 questions. **Show all work!** Answers with no justification will receive no points. Please begin each problem on a new page. No notes, calculators, or electronic devices are permitted.

1. (18 points) Find the requested information.

(a) $f(x) = x^{\cos x}(1 + e^x)$, find $f'(x)$

(b) Find dy/dx and d^2y/dx^2 at the point $(1, 0)$ for the equation $x^2y + 2x = 2 - \tan y$.

(c) Find the value (in terms of n) of $\sum_{i=1}^n (3i + 4)$.

2. (18 points) Find the requested information.

(a) $\int (x^5 + 5^x) dx$

(b) $\int \frac{\log_8 x}{x} dx$

(c) Find the average value of $g(x) = \tan x$ on the interval $[0, \pi/4]$.

3. (30 points) Find the requested limit, if it exists. If the limit does not exist, please explain why it doesn't exist.

(a) $\lim_{u \rightarrow 2} \frac{2 + u}{u^2 - 4}$

(b) $\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x}\right)^{bx}$

(c) $\lim_{x \rightarrow \infty} \frac{\sinh(x)}{e^x}$

(d) $\lim_{x \rightarrow 0^+} \tan^{-1}(\ln x)$

(e) $\lim_{h \rightarrow 0} \frac{1}{h} \int_3^{3+h} \sqrt{1 + t^2} dt$

4. (24 points) Let $f(x) = x^2e^{-x}$. Justify your answers in each of the following parts.

(a) Does $f(x)$ have any vertical or horizontal asymptotes? If so, find them. If not, state this.

(b) Find the interval(s) on which $f(x)$ is increasing and the interval(s) on which it is decreasing. Give the x -coordinates of any local maximums or minimums.

(c) Find the interval(s) on which $f(x)$ is concave up and the intervals on which it is concave down. Give the x -coordinates of any inflection points.

(d) Using the information in parts(a)-(c), carefully sketch the graph of $f(x)$. Be sure to clearly label any asymptotes and the x -coordinates for any maximums, minimums, and inflection points.

5. (15 points) Let $f(x) = ax^2 + bx + c$ and the graph goes through the point $(1, 2)$. At the origin, $f(x)$ and $g(x) = \sin(x) + x^{5/3}$ have the same tangent line. Use this information to find a , b , and c . Explain your work.
6. (15 points) Let $h(x) = \frac{1}{x^{2/3}} + \int_1^{x^2} \frac{3t}{t^4 + 2} dt$
- (a) Find the linearization of $h(x)$ at the point $x = -1$.
 - (b) Approximate $h(-1/2)$ using your linearization.
7. (15 points) A ladder 10 ft long leans against a vertical wall. If the bottom of the ladder slides away from the base of the wall at a speed of 2 ft/s, how fast is the angle between the ladder and the wall changing when the bottom of the ladder is 6 ft from the base of the wall?
8. (15 points) What is the area of the largest rectangle in the first quadrant, under the curve $y = e^{-x}$, with two sides on the axes and one vertex on the curve $y = e^{-x}$?