1. $f(x) = x^{\cos x}(1+e^x) = y$

$$\ln y = (\cos x)(\ln x) + \ln(1+e^x)$$

$$\frac{y'}{y} = -(\sin x)(\ln x) + \frac{\cos x}{x} + \frac{e^x}{1+e^x}$$

$$y' = y\left[\frac{\cos x}{x} + \frac{e^x}{1+e^x} - \sin x \ln x\right]$$

$$= x^{\cos x}(1+e^x)\left(\frac{\cos x}{x} + \frac{e^x}{1+e^x} - \sin x \ln x\right) \checkmark$$

b. $x^2 y + 2x = 2 - \tan y$

$$2x\,dx \cdot y + x^2\,dy + 2\,dx = -\sec^2 y\,dy$$

$$(2xy + 2)\,dx = (-x^2 - \sec^2 y)\,dy$$

$$\frac{2xy+2}{-x^2-\sec^2 y} = \frac{dy}{dx} \Rightarrow \left.\frac{dy}{dx}\right]_{(1,0)} = \frac{2}{-2} = -1 \checkmark$$

$$\frac{d^2y}{dx^2} = \frac{(2(y+xy'))\cdot(-x^2-\sec^2 y) - (2xy+2)(-2x-2\sec^2 y\tan y\cdot y')}{(-x^2-\sec^2 y)^2}$$

$$\left.\frac{d^2y}{dx^2}\right]_{(1,0)} = \frac{(2(-1))\cdot(-2) - (2)(-2-2(1)(0)(-1))}{4} = 2 \checkmark$$

c. $$\sum_{i=1}^{n}(3i+4) = 3\sum_{i=1}^{n} i + 4\sum_{i=1}^{n} 1 = \frac{3(n)(n+1)}{2} + 4n = \frac{3n^2+3n+8n}{2}$$

$$= \frac{3n^2+11n}{2} \checkmark$$

a. $\int x^5 + 5^x \, dx = \frac{x^6}{6} + \frac{5^x}{\ln 5} + C$ ✓

b. $\int \frac{\log_8 x}{x} \, dx = \int \frac{\ln x}{\ln 8} \cdot \frac{1}{x} \, dx$

Let $u = \ln x$
$du = dx/x$

$= \int \frac{u}{\ln 8} \, du = \frac{u^2}{2 \ln 8} + C = \frac{(\ln x)^2}{2 \ln 8} + C$ ✓

c. $AVG(g(x)) = \frac{1}{\frac{\pi}{4} - 0} \int_0^{\frac{\pi}{4}} \tan x \, dx$

$= \frac{4}{\pi} \left[-\ln(\cos x) \right]_0^{\pi/4} = -\frac{4}{\pi} \left[\ln(\sqrt{2}/2) - 0 \right] = -\frac{4}{\pi} \cdot \frac{-1}{2} \ln(2)$

$= \frac{\ln(4)}{\pi}$ ✓

3. $\lim_{u\to 2} \frac{u+2}{u^2-4} = \lim_{u\to 2} \frac{\cancel{u+2}}{\cancel{(u+2)}(u-2)} = \lim_{u\to 2} \frac{1}{u-2} = DNE$

because $\lim_{u\to 2^+} \frac{1}{u-2} = \infty \neq -\infty = \lim_{u\to 2^-} \frac{1}{u-2}$. ✓

b. $\lim_{x\to\infty} \left(1+\frac{a}{x}\right)^{bx}$, assuming $a>0$, let $t = \frac{x}{a} \Rightarrow at = x$
$\Rightarrow t \to \infty$ as $x \to \infty$

$= \lim_{t\to\infty} \left(1+\frac{1}{t}\right)^{b(at)} = \lim_{t\to\infty} \left[\left(1+\frac{1}{t}\right)^t\right]^{ab} = \left[\lim_{t\to\infty}\left(1+\frac{1}{t}\right)^t\right]^{ab} = e^{ab}$ ✓

c. $\lim_{x\to\infty} \frac{\sinh(x)}{e^x} = \lim_{x\to\infty} \frac{e^x - e^{-x}}{2}\cdot\frac{1}{e^x}$

$= \lim_{x\to\infty} \frac{(e^x - e^{-x})e^{-x}}{2} = \lim_{x\to\infty} \frac{1-e^{-2x}}{2} = \frac{1}{2}$ ✓

d. $\lim_{x\to 0^+} \tan^{-1}(\ln x) = \lim_{t\to -\infty} \tan^{-1}(t) = -\pi/2$ ✓

e. $\lim_{h\to 0} \frac{\int_3^{3+h} \sqrt{1+t^2}\, dt}{h} = \lim_{h\to 0} \frac{F(3+h) - F(3)}{h} = F'(3)$

$= \sqrt{1+t^2}\Big]_{t=3} = \sqrt{10}$ where $F(t)$ is any anti-derivative
of $\sqrt{1+t^2} \Rightarrow F'(t) = \sqrt{1+t^2}$. ✓

4. $f(x) = x^2 e^{-x}$

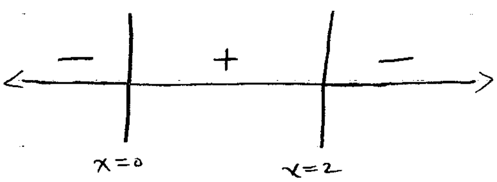
a. No V.A. because Domain $(f(x))$ = all real numbers

$\lim_{x\to\infty} x^2e^{-x} = \lim_{x\to\infty} \frac{x^2}{e^x} = 0$, $\lim_{x\to-\infty} x^2e^{-x} = \lim_{t\to\infty} (-t)^2 e^{-(-t)}$

So only one H.A. at $y=0$. $= \lim_{t\to\infty} t^2e^t = \infty$ where $t=-x$.

b. $f'(x) = 2xe^{-x} - x^2e^{-x} = 0 \Rightarrow 2x\cancel{e^{-x}} = x^2\cancel{e^{-x}}$ because $e^{-x} \neq 0$

$\Rightarrow x^2 - 2x = 0 \Rightarrow x(x-2) = 0 \Rightarrow x = 0, 2$ critical points



$f'(-1) < 0$, $f'(1) > 0$, $f'(3) < 0$

f decreasing on $(-\infty, 0) \cup (2, \infty)$

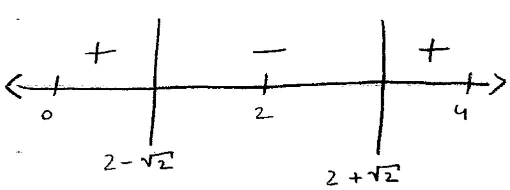
f increasing on $(0, 2)$

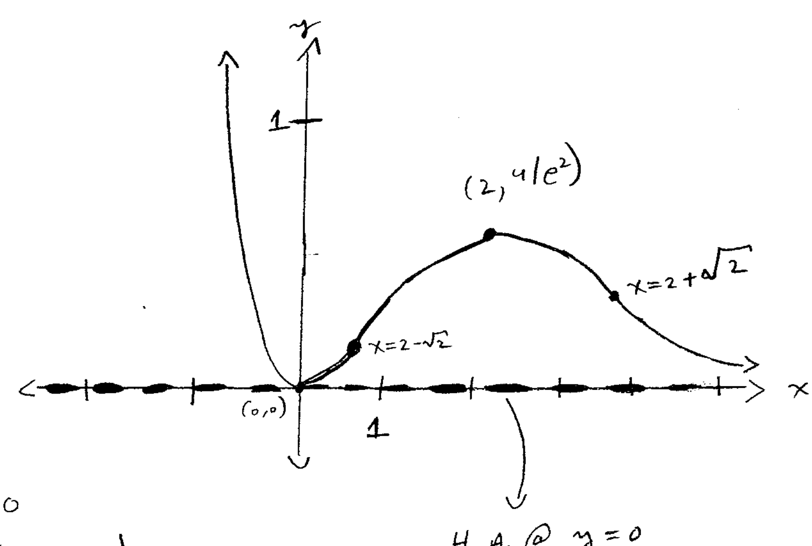
Max at $x = 2$, min at $x = 0$

c. $f''(x) = 2e^{-x} - 2xe^{-x} - 2xe^{-x} + x^2e^{-x} = (2 - 4x + x^2)e^{-x} = 0$

$\Rightarrow x^2 - 4x + 2 = 0$

$\Rightarrow x = \frac{4 \pm \sqrt{8}}{2} = 2 \pm \sqrt{2}$





$f''(0) > 0$, $f''(2) < 0$, $f(4) > 0$

f is C.U. on $(-\infty, 2-\sqrt{2}) \cup (2+\sqrt{2}, \infty)$

f is C.D. on $(2-\sqrt{2}, 2+\sqrt{2})$

5. $f(x) = ax^2 + bx + c \Rightarrow f'(x) = 2ax + b$

goes through $(1,2) \Rightarrow 2 = a + b + c$

goes through $(0,0) = \quad 0 = c$

$g(x) = \sin x + x^{5/3} \Rightarrow g'(x) = \cos x + \frac{5}{3} x^{2/3}$

$\Rightarrow g'(0) = 1$

$\Rightarrow f'(0) = 1 = b$

$\Rightarrow a = 1, \; b = 1, \; c = 0$ ✓

6. $h(x) = x^{-2/3} + \int_1^{x^2} \frac{3t}{t^4+2}\, dt$

$h(-1) = (-1)^{-2/3} + \int_1^1 \frac{3t}{t^4+2}\, dt = +1$

$h'(x) = (-2/3)\, x^{-5/3} + \frac{3x^2}{x^8+2} \cdot 2x \Rightarrow h'(-1) = 2/3 + \frac{3(-2)}{3} = \frac{2}{3} - 2 = -\frac{4}{3}$

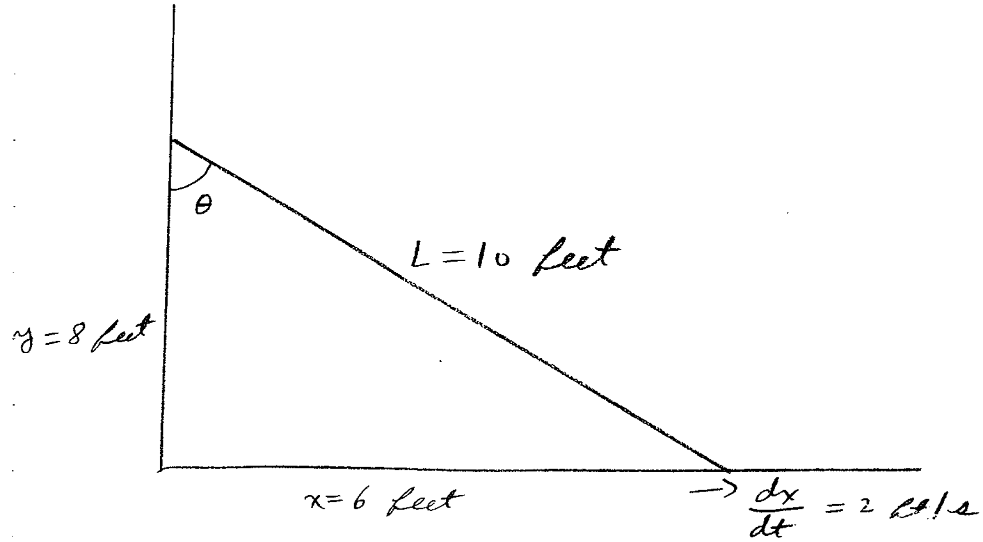
$\Rightarrow y - (+1) = -\frac{4}{3}(x - (-1)) \Rightarrow y - 1 = -\frac{4}{3}(x+1) \Rightarrow y = -\frac{4}{3}x - \frac{1}{3}$

$h(-1/2) \approx y(-1/2) = -\frac{4}{3}(-1/2) - \frac{1}{3} = 4/6 - 1/3 = 4/6 - 2/6$

$= 2/6$

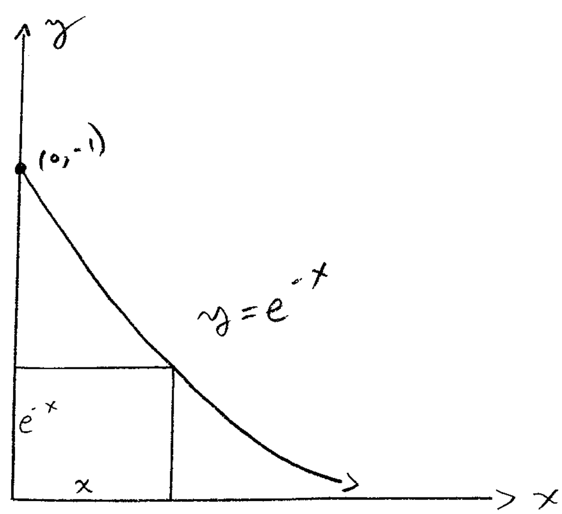
$= 1/3$ ✓

7.



$$\sin\theta = \frac{x}{L} \Rightarrow \cos\theta \frac{d\theta}{dt} = \frac{1}{L}\frac{dx}{dt} \Rightarrow \frac{8}{10}\frac{d\theta}{dt} = \frac{2}{10} \Rightarrow \frac{d\theta}{dt} = \frac{1}{4}$$ ✓

8.



$$A = l \cdot h = xe^{-x} \Rightarrow A'(x) = e^{-x} - xe^{-x} = 0 \Rightarrow 1 - x = 0 \Rightarrow x = 1$$

$$A''(x) = -e^{-x} - e^{-x} + xe^{-x} = -2e^{-x} + xe^{-x} \Rightarrow A''(1) = -e^{-1} < 0$$

so at $x = 1$ we have a max.

Max area possible is $A(1) = e^{-1} = 1/e$. ✓