**INSTRUCTIONS:** Books, notes, and electronic devices are not permitted. Write (1) **your name**, (2) **1350/FINAL**, (3) **instructor's name** and (4) **SPRING 2011** on the front of your bluebook. Also make a scoring table with room for 8 problems and a total score. **Work all problems. Start each problem on a new page.** **Box** **your answers.** A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

**— SHOW ALL WORK —**

1. (24 pts - 6pts ea) Assume $y$ is a function of $x$, find $y'$ given:

   (a) $y = \dfrac{1}{\ln(x)}$ (b) $xe^{y^2} = \tan^{-1}(e^x) - y$ (c) $y = (\cos x)^x$ (d) $y = \displaystyle\int_e^{e^x} (t)^{\ln(t)}\, dt$

2. (24 pts - 6pts ea) Evaluate the integrals:

   (a) $\displaystyle\int \frac{\ln(x^2 e^{\sqrt{x}})}{x}\, dx$ (b) $\displaystyle\int \frac{dx}{x^{3/2} + x^{1/2}}$ (c) $\displaystyle\int_0^4 \frac{x}{\sqrt{1+2x}}\, dx$ (d) $\displaystyle\int_0^1 \ln(\sinh(x) + \cosh(x))\, dx$

3. (24 pts - 6pts ea) Find the limits

   (a) $\lim_{x\to\infty} (1-2x)^{1/x}$ (b) $\lim_{x\to-\infty} \dfrac{x}{\sqrt{1+x^2}}$ (c) $\lim_{x\to\infty} \dfrac{1}{2\tan^{-1}(x) - \pi}$ (d) $\lim_{x\to e} \dfrac{1}{x-e}$

4. (a) (5 pts) Find the linearization of $f(x) = \ln(1-x)$ at $a = 0$.

   (b) (5 pts) Use your linearization from part (a) to approximate $\ln(0.99)$.

5. (a) (5 pts) Consider a cylinder with a circular base and a fixed height of 7 inches. Suppose the radius of the base is $r$. If V is the volume of the cylinder, find the rate of change of volume in terms of the rate of change of the radius $r$ with respect to time. (Note, $V = 7\pi r^2$)

   (b) (10 pts) Suppose the radius of the cylinder mentioned in part (a) is measured to be 0.8 inches with an error in measurement of 0.01 inches, use differentials to estimate the percentage error in calculating the volume of the cylinder.

6. The velocity function of a particle moving along a straight line is given by $v(t) = t^2 + 3t - 4$ m/s with initial position $s(0) = 106$ m.

   (a) (5 pts) Find the position of the particle at any time t

   (b) (5 pts) Find the acceleration of the particle at any time t

   (c) (10 pts) Find the total distance travelled by the particle during the first 3 seconds.

7. (10 pts) Use the Mean Value Theorem to prove that if $a < b$ then $\sin(b) - \sin(a) \le b - a$.

8. Given that $f(x) = \dfrac{x^2}{2} + \dfrac{\ln[(x-2)^2]}{2}$, $f'(x) = x + \dfrac{1}{(x-2)}$, and $f''(x) = 1 - \dfrac{1}{(x-2)^2}$

   (a) (2 pts) State the domain of $f(x)$.

   (b) (4 pts) Does $f(x)$ have any *vertical asymptotes*? Justify your answer with a limit.

   (c) (4 pts) Does $f(x)$ have any *horizontal asymptotes*? Justify your answer with a limit.

   (d) (4 pts) On what interval (or intervals) is $f(x)$ *increasing* and/or *decreasing*? Justify your answer.

   (e) (4 pts) On what interval (or intervals) is $f(x)$ *concave up* and/or *concave down*? Justify your answer.

   (f) (2 pts) Find all local maximum or minimum values of $f(x)$.

   (g) (3 pts) Sketch the graph of $f(x) = \dfrac{x^2}{2} + \dfrac{\ln[(x-2)^2]}{2}$. (Clearly label and sketch your graph.)

END