APPM 1350 **Final Exam** **Summer 2011**

CAREFULLY PRINT, on the front of your bluebook: a grading key, your name, student ID, section, and instructor's name (Guinn). This exam is worth 150 points (note that there are 165 possible points meaning up to 15 points of extra credit is possible) and has 7 questions. **Show all work!** Answers with no justification will receive no points. Please begin each problem on a new page. No notes, calculators, or electronic devices are permitted.

1. (30 points) Find **THREE** of the following limits, if they exist. Make sure to provide justification as to why each limit does or does not exist. Please clearly mark which three limits you would like to be graded.

   (a) $\lim_{x\to\infty} x^{\ln\frac{1}{9}}(1+\ln x)$

   (b) $\lim_{n\to\infty} \left(1+\dfrac{r}{n}\right)^{nt}$

   (c) $\lim_{x\to 0} \ln\left(\tan^{-1}(x)\right)$

   (d) $\lim_{h\to 0} \dfrac{1}{h}\displaystyle\int_1^{1+h} \sqrt{1+t^2}\,dt$

2. (20 points) Find $\dfrac{dy}{dx}$ for **TWO** of the following functions. Do NOT simplify your answer. Please clearly indicate which two derivatives you would like to be graded.

   (a) $y = 2^{\cosh(x)}(\ln x^2)$

   (b) $x^2 y = e^{xy} - \log_3(\cos x)$

   (c) $y = \dfrac{x(x^2-2)^{3/2}}{\sqrt{x+1}(x^3+4)}$

3. (20 points) Answer **TWO** of the following three problems. Please clearly indicate which two problems you would like to be graded.

   (a) Evaluate $\displaystyle\int x3^{x^2}\,dx$.

   (b) Find the average value of $f(x) = \dfrac{\log_2 x}{x}$ on the interval $[1,4]$.

   (c) Find the linearization of $\displaystyle\int_1^x \frac{1}{3+t+t^2}\,dt$ near $x=1$.

4. (30 points) A boy flies a kite at a height of 400 ft. The wind blows the kite horizontally away from him at a rate of 20 ft/sec. How fast is the angle between the ground and the kite string changing when there is 500 ft of kite string out?

5. (25 points) What is the area of the largest rectangle in the first quadrant, under the curve $y = e^{-x}$, with two sides on the axes and one vertex on the curve $y = e^{-x}$?

TURN PAGE OVER FOR PROBLEMS #6-7

6. (25 points) Carbon-14 (^{14}C) is a radioactive element found in organic (living) material, and is used to determine the age of archaeological samples. ^{14}C decays at a rate proportional to its mass. This means that at any time t, the amount of ^{14}C remaining from an initial sample of mass m_0 is given by: $m(t) = m_0 e^{kt}$, where k is the relative decay constant.

   (a) If ^{14}C has a relative decay constant of $k \approx -1.2\text{x}10^{-4} = -0.00012$, find a formula for the half-life of ^{14}C. Your formula should have only constant values in it, but you do not need to find an actual number.

   (b) Find a formula for the number of years until there are 15 grams of ^{14}C remaining from an original 100 gram sample. About how many half-lives is this?

7. (15 points) TRUE/FALSE. For this problem only, just write TRUE or FALSE, no explanation is required.

   (a) If $\lim_{x\to a} f(x) = \lim_{x\to a} g(x)$, then $f(a) = g(a)$.

   (b) If $f(x) \leq g(x)$ for all $a \leq x \leq b$, then $\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx$.

   (c) If $f(x) \leq g(x)$ for all $a \leq x \leq b$, then $f'(x) \leq g'(x)$ for all $a \leq x \leq b$.

   (d) If $h(x)$ is differentiable, then $h(x)$ is continuous.

   (e) $\lim_{x\to 1} \log_{10} x = 0$.