— Useful and interesting formulae —

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \qquad \cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$V = \int_a^b \pi \left[f(x)\right]^2 \, dx = \int_c^d 2\pi y f(y)\, dy \qquad A = \int_a^b 2\pi f(x) \sqrt{1 + \left(\frac{df}{dx}\right)^2}\, dx$$

$$L = \int_a^b \sqrt{1 + \left(\frac{df}{dx}\right)^2}\, dx = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_{\theta_1}^{\theta_2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$M = \int dm, \qquad M_x = \int \tilde{y}\, dm, \qquad M_y = \int \tilde{x}\, dm, \qquad (\bar{x}, \bar{y}) = \left(\frac{M_y}{M}, \frac{M_x}{M}\right)$$

$$A = \int_{\theta_1}^{\theta_2} \frac{[f(\theta)]^2}{2}\, d\theta$$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C \qquad \int \frac{du}{\sqrt{a^2 + u^2}} = \sinh^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1}\left(\frac{u}{a}\right) + C \qquad \int \frac{du}{a^2 - u^2} = \frac{1}{a} \tanh^{-1}\left(\frac{u}{a}\right) + C \quad (u^2 < a^2)$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1}\left|\frac{u}{a}\right| + C \qquad \int \frac{du}{u\sqrt{a^2 - u^2}} = -\frac{1}{a} \operatorname{sech}^{-1}\left|\frac{u}{a}\right| + C$$

$$\int \frac{du}{\sqrt{u^2 - a^2}} = \cosh^{-1}\left(\frac{u}{a}\right) + C \qquad \int \frac{du}{u\sqrt{u^2 + a^2}} = -\frac{1}{a} \operatorname{csch}^{-1}\left|\frac{u}{a}\right| + C$$

$$\int \sec(u)\, du = \ln|\sec(u) + \tan(u)| + C \qquad \int \csc(u)\, du = -\ln|\csc(u) + \cot(u)| + C$$

$$\int \sqrt{a^2 + u^2}\, du = \frac{u}{2}\sqrt{a^2 + u^2} + \frac{a^2}{2} \ln\left|u + \sqrt{a^2 + u^2}\right| + C$$

$$\int \sqrt{a^2 - u^2}\, du = \frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1}\left(\frac{u}{a}\right) + C$$

$$\lim_{n\to\infty} \frac{\ln n}{n} = 0 \qquad \lim_{n\to\infty} n^{(1/n)} = 1$$

$$\lim_{n\to\infty} x^{(1/n)} = 1 \ \text{(for } x > 0\text{)} \qquad \lim_{n\to\infty} x^n = 0 \ \text{(for } |x| < 1\text{)}$$

$$\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = e^x \ \text{(for any } x\text{)} \qquad \lim_{n\to\infty} \frac{x^n}{n!} = 0 \ \text{(for any } x\text{)}$$

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r} \text{ if } |r| < 1$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \qquad \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots \qquad \cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots \qquad \tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$$

$$(1+x)^m = 1 + mx + \frac{m(m-1)}{2!}x^2 + \ldots + \frac{m(m-1)(m-2)\ldots(m-n+1)}{n!}x^n + \ldots$$

$$\text{Ellipse: } c = \sqrt{|a^2 - b^2|}, \qquad \text{Hyperbola: } c = \sqrt{a^2 + b^2}$$

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0 :$$

$$\tan(2\theta) = \frac{B}{A-C} \Leftrightarrow \cot(2\theta) = \frac{A-C}{B}$$

$$B^2 - 4AC \begin{cases} < 0 & \Rightarrow \text{ellipse} \\ = 0 & \Rightarrow \text{parabola} \\ > 0 & \Rightarrow \text{hyperbola} \end{cases}$$

$$\begin{aligned} A' &= A\cos^2(\theta) + B\cos(\theta)\sin(\theta) + C\sin^2(\theta) \\ B' &= B\cos(2\theta) + (C-A)\sin(2\theta) \\ C' &= A\sin^2(\theta) - B\cos(\theta)\sin(\theta) + C\cos^2(\theta) \\ D' &= D\cos(\theta) + E\sin(\theta) \\ E' &= -D\sin(\theta) + E\cos(\theta) \\ F' &= F \end{aligned}$$

$$\left.\begin{aligned} x &= \cos(\theta)x' - \sin(\theta)y' \\ y &= \sin(\theta)x' + \cos(\theta)y' \end{aligned}\right\} \Leftrightarrow \left\{\begin{aligned} x' &= \cos(\theta)x + \sin(\theta)y \\ y' &= -\sin(\theta)x + \cos(\theta)y \end{aligned}\right.$$