INSTRUCTIONS: Books, notes, and electronic devices are not permitted. Write your (1) name, (2) instructor's name, and (3) when your lecture meets on the front of your bluebook. Also make a scoring table, with places for 5 problems plus a total score. **Work all problems. Start each problem on a new page.** Box your answers. A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **SHOW ALL WORK.**

For problems 1 through 3, consider the region in the first quadrant bounded above by the function $y = \sqrt{x}$ and below by the function $y = \dfrac{x^2}{8}$.

1. (15 points)

   (a) Set up, but DO NOT EVALUATE, the integral(s) needed to compute the perimeter of the region.

   (b) Find the total area of the region.

2. (25 points) Find the center of mass of the region, assuming a constant density δ.

3. (16 points) Set up, but DO NOT EVALUATE, the integral(s) needed to compute the following:

   (a) The volume of the solid generated by revolving the region around the y-axis. Use the washer method.

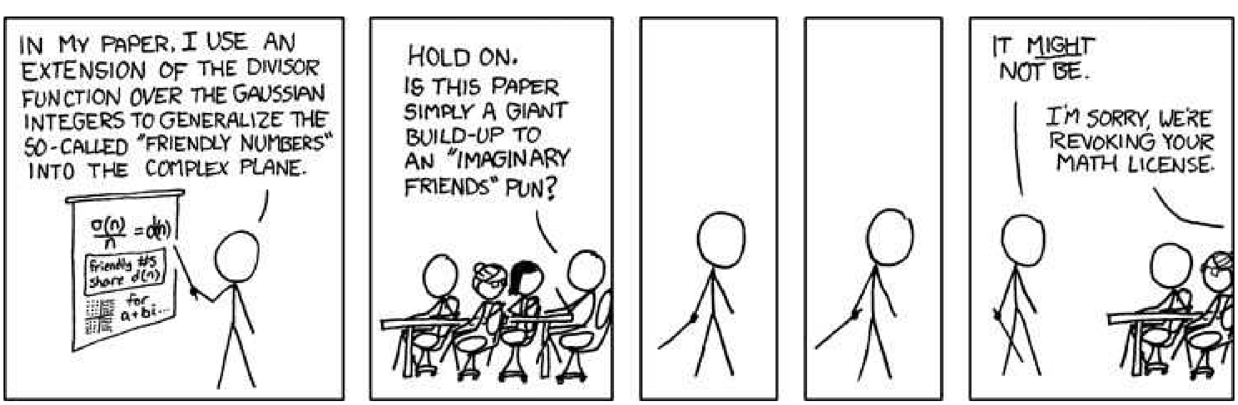
   (b) The volume of the solid generated by revolving the region around the line $y = -2$. Use the shell method.

4. (24 points) Compute the following integrals (simplify your answer as much as possible):

   (a) $\displaystyle\int_1^{e^2} \ln x\, dx$ (b) $\displaystyle 2\int_0^{\pi/4} \frac{\sin x + \cos^2 x}{\cos x} dx$ (c) $\displaystyle\int \frac{1}{\cosh x + 1} dx$

5. (20 points) Find a function $y(x)$ such that $y(1) = 0$ and

$$(x^2 + 2x)\frac{dy}{dx} = 2y + 2$$



There are helpful formulae on the back!

Formulae

The following equations may be useful

- Equations from Chapter 5
  Note: these are all definite integrals; the limits of integration and the variable you integrate with respect to depend on the formulation of the problem!

  1. Volume by Slicing
     $$V = \int A(\cdot)d\cdot$$
  2. Volume of Revolution (Disks / Washers):
     $$V = \int \pi\Big([R(\cdot)]^2 - [r(\cdot)]^2\Big)d\cdot$$
  3. Volume of Revolution (Shells):
     $$V = \int 2\pi r(\cdot)h(\cdot)d\cdot$$
  4. Length of a Plane curve:
     $$L = \int ds$$
  5. Surface Area of Revolution:
     $$S = \int 2\pi\rho\, ds$$
  6. Moments and Center of Mass (Rod):
     $$M = \int dm$$
     $$M_0 = \int x\, dm$$
     $$\bar{x} = \frac{M_0}{M}$$
  7. Moments and Center of Mass (Region):
     $$M = \int dm$$
     $$M_x = \int \tilde{y}\, dm$$
     $$M_y = \int \tilde{x}\, dm$$
     $$\bar{x} = \frac{M_y}{M},\ \bar{y} = \frac{M_x}{M}$$

- Integration table ($a \neq 0$):

$$\int \sinh u\, du = \cosh u + C$$
$$\int \cosh u\, du = \sinh u + C$$
$$\int \operatorname{sech}^2 u\, du = \tanh u + C$$
$$\int \operatorname{csch}^2 u\, du = -\coth u + C$$
$$\int \operatorname{sech} u \tanh u\, du = -\operatorname{sech} u + C$$
$$\int \operatorname{csch} u \coth u\, du = -\operatorname{csch} u + C$$

$$\int \frac{du}{\sqrt{a^2+u^2}} = \sinh^{-1}\left(\frac{u}{a}\right) + C, \qquad a > 0$$
$$\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}\left(\frac{u}{a}\right) + C, \qquad u > a > 0$$
$$\int \frac{du}{a^2-u^2} = \begin{cases} \dfrac{1}{a}\tanh^{-1}\left(\dfrac{u}{a}\right) + C, & u^2 < a^2 \\ \dfrac{1}{a}\coth^{-1}\left(\dfrac{u}{a}\right) + C, & u^2 > a^2 \end{cases}$$
$$\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\operatorname{sech}^{-1}\left(\frac{u}{a}\right) + C, \qquad 0 < u < a$$
$$\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\operatorname{csch}^{-1}\left(\frac{u}{a}\right) + C, \qquad u \neq 0$$