INSTRUCTIONS: Books, notes, and electronic devices are not permitted. Write (1) your name, (2) instructor's name, and (3) when your lecture meets on the front of your bluebook. Also make a scoring table, with places for 6 problems, plus a total score. **Work all problems. Start each problem on a new page. Clearly mark your answers.** A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

SHOW ALL WORK.

1. (16 points) Determine if the given series converges conditionally, absolutely or diverges:

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n^2}$ (b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$

2. (20 points)

   (a) Find the Maclaurin Series of $f(x) = \ln(1+x)$. SHOW ALL WORK!

   (b) Use the first three nonzero terms of the series found in part (a) to estimate $\displaystyle\int_0^{0.1} \ln(1+x)\, dx$.

   (c) Estimate the error of the approximation used in part (b).

   (d) Is the approximation used in part (b) an underestimate or an overestimate?

3. (20 points)

   (a) Find the radius of convergence of: $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x+2)^n}{n2^n}$.

   (b) For what value(s) of x does the series in part (a) converge absolutely?

   (c) For what value(s) of x does the series in part (a) converge conditionally but not absolutely?

   (d) For what value(s) of x does the series in part (a) diverge?

4. (16 points)

   (a) Find the Maclaurin Series for $f(x) = 2xe^{x^2}$. (You may use your knowledge of the Maclaurin Series of e^x to answer this question.)

   (b) Integrate the series term by term over the interval $[0, 1]$ and thus, or otherwise, show

   $$\sum_{n=0}^{\infty} \frac{1}{(n+1)n!} = e - 1$$

5. (16 points)

   (a) Use series to evaluate: $\displaystyle\lim_{x\to 0} \frac{x - \tan^{-1}(x)}{x^3}$.

   (b) Find the first four terms of the binomial series representation of $f(x) = (1 + 1/x)^{1/2}$.

6. (12 points) Classify the equation (circle, ellipse, hyperbola, etc.)

   $$\frac{x^2}{c-9} + \frac{y^2}{16-c} = 1$$

   for the values of c given below:

   (a) $c < 9$ (b) $9 < c < 16$ (c) $c > 16$

THERE ARE SOME FORMULAE ON THE OTHER SIDE.

FREQUENTLY USED MACLAURIN SERIES:

- $\dfrac{1}{1-x} = \displaystyle\sum_{n=0}^{\infty} x^n, \quad |x| < 1$
- $\dfrac{1}{1+x} = \displaystyle\sum_{n=0}^{\infty} (-1)^n x^n, \quad |x| < 1$
- $e^x = \displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad |x| < \infty$
- $\sin(x) = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \quad |x| < \infty$
- $\cos(x) = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \quad |x| < \infty$
- $\tan^{-1}(x) = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}, \quad |x| \leq 1$
- $(1+x)^m = 1 + \displaystyle\sum_{k=1}^{\infty} \binom{m}{k} x^k, \quad |x| < 1$