

Formulae

The following equations may be useful

- Equations from Chapter 5

Note: these are all definite integrals; the limits of integration and the variable you integrate with respect to depend on the formulation of the problem!

1. Volume by Slicing

$$V = \int A(\cdot) d\cdot$$

2. Volume of Revolution (Disks / Washers):

$$V = \int \pi \left([R(\cdot)]^2 - [r(\cdot)]^2 \right) d\cdot$$

3. Volume of Revolution (Shells):

$$V = \int 2\pi r(\cdot) h(\cdot) d\cdot$$

4. Length of a Plane curve:

$$L = \int ds$$

5. Surface Area of Revolution:

$$S = \int 2\pi \rho ds$$

6. Moments and Center of Mass (Rod):

$$M = \int dm$$

$$M_0 = \int x dm$$

$$\bar{x} = \frac{M_0}{M}$$

7. Moments and Center of Mass (Region):

$$M = \int dm$$

$$M_x = \int \tilde{y} dm$$

$$M_y = \int \tilde{x} dm$$

$$\bar{x} = \frac{M_y}{M}, \bar{y} = \frac{M_x}{M}$$

- Integration table ($a \neq 0$):

$$\int \sinh u du = \cosh u + C$$

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$$\int \operatorname{sech}^2 u du = \tanh u + C$$

$$\int \operatorname{csch}^2 u du = -\operatorname{coth} u + C$$

$$\int \operatorname{sech} u \tanh u du = -\operatorname{sech} u + C$$

$$\int \operatorname{csch} u \operatorname{coth} u du = -\operatorname{csch} u + C$$

$$\int \frac{du}{\sqrt{a^2 + u^2}} = \sinh^{-1} \left(\frac{u}{a} \right) + C, \quad a > 0$$

$$\int \frac{du}{\sqrt{u^2 - a^2}} = \cosh^{-1} \left(\frac{u}{a} \right) + C, \quad u > a > 0$$

$$\int \frac{du}{a^2 - u^2} = \begin{cases} \frac{1}{a} \tanh^{-1} \left(\frac{u}{a} \right) + C, & u^2 < a^2 \\ \frac{1}{a} \operatorname{coth}^{-1} \left(\frac{u}{a} \right) + C, & u^2 > a^2 \end{cases}$$

$$\int \frac{du}{u\sqrt{a^2 - u^2}} = -\frac{1}{a} \operatorname{sech}^{-1} \left(\frac{u}{a} \right) + C, \quad 0 < u < a$$

$$\int \frac{du}{u\sqrt{a^2 + u^2}} = -\frac{1}{a} \operatorname{csch}^{-1} \left(\frac{u}{a} \right) + C, \quad u \neq 0$$