Formulae

The following equations may be useful

- Equations from Chapter 5
  Note: these are all definite integrals; the limits of integration and the variable you integrate with respect to depend on the formulation of the problem!

1. Volume by Slicing
$$V = \int A(\cdot) d\cdot$$
2. Volume of Revolution (Disks / Washers):
$$V = \int \pi\Big([R(\cdot)]^2 - [r(\cdot)]^2\Big) d\cdot$$
3. Volume of Revolution (Shells):
$$V = \int 2\pi r(\cdot) h(\cdot) d\cdot$$
4. Length of a Plane curve:
$$L = \int ds$$
5. Surface Area of Revolution:
$$S = \int 2\pi \rho \, ds$$
6. Moments and Center of Mass (Rod):
$$M = \int dm$$
$$M_0 = \int x \, dm$$
$$\bar{x} = \frac{M_0}{M}$$
7. Moments and Center of Mass (Region):
$$M = \int dm$$
$$M_x = \int \tilde{y} \, dm$$
$$M_y = \int \tilde{x} \, dm$$
$$\bar{x} = \frac{M_y}{M}, \; \bar{y} = \frac{M_x}{M}$$

- Integration table ($a \neq 0$):

$$\int \sinh u \, du = \cosh u + C$$
$$\int \cosh u \, du = \sinh u + C$$
$$\int \operatorname{sech}^2 u \, du = \tanh u + C$$
$$\int \operatorname{csch}^2 u \, du = -\coth u + C$$
$$\int \operatorname{sech} u \tanh u \, du = -\operatorname{sech} u + C$$
$$\int \operatorname{csch} u \coth u \, du = -\operatorname{csch} u + C$$
$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C \quad u^2 < a^2$$
$$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$
$$\int \frac{du}{\sqrt{a^2 + u^2}} = \sinh^{-1}\left(\frac{u}{a}\right) + C, \quad a > 0$$
$$\int \frac{du}{\sqrt{u^2 - a^2}} = \cosh^{-1}\left(\frac{u}{a}\right) + C, \quad u > a > 0$$
$$\int \frac{du}{a^2 - u^2} = \begin{cases} \dfrac{1}{a}\tanh^{-1}\left(\dfrac{u}{a}\right) + C, & u^2 < a^2 \\ \dfrac{1}{a}\coth^{-1}\left(\dfrac{u}{a}\right) + C, & u^2 > a^2 \end{cases}$$
$$\int \frac{du}{u\sqrt{a^2 - u^2}} = -\frac{1}{a}\operatorname{sech}^{-1}\left(\frac{u}{a}\right) + C, \quad 0 < u < a$$
$$\int \frac{du}{u\sqrt{a^2 + u^2}} = -\frac{1}{a}\operatorname{csch}^{-1}\left(\frac{u}{a}\right) + C, \quad u \neq 0$$
$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{u}{a}\right) + C, \quad u^2 > a^2$$

There are some more formulas on the back of this page!

- Trig and Hyperbolic identities

$\cos^2 x + \sin^2 x = 1$

$\cos^2 x = \dfrac{1+\cos(2x)}{2}$

$\sin^2 x = \dfrac{1-\cos(2x)}{2}$

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cosh^2 x - \sinh^2 x = 1$

$\cosh^2 x = \dfrac{\cosh(2x)+1}{2}$

$\sinh^2 x = \dfrac{\cosh(2x)-1}{2}$

$\sinh 2x = 2\sinh x \cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

- Common Limits

$\lim_{n\to\infty} \dfrac{\ln n}{n} = 0$

$\lim_{n\to\infty} \sqrt[n]{n} = 1$

$\lim_{n\to\infty} \sqrt[n]{x} = 1 \quad x > 0$

$\lim_{n\to\infty} x^n = 0 \quad |x| < 1$

$\lim_{n\to\infty} \left(1 + \dfrac{x}{n}\right)^n = e^x \quad x \in \mathcal{R}$

$\lim_{n\to\infty} \dfrac{x^n}{n!} = 0 \quad x \in \mathcal{R}$

- Common Series

$\dfrac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad |x| < 1$

$e^x = \sum_{n=0}^{\infty} \dfrac{x^n}{n!}, \quad x \in \mathcal{R}$

$\sin x = \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{2n+1}}{(2n+1)!}, \quad x \in \mathcal{R}$

$\cos x = \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{2n}}{(2n)!}, \quad x \in \mathcal{R}$

$\tan^{-1} x = \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{2n+1}}{2n+1}, \quad |x| \le 1$

$(1+x)^m = 1 + \sum_{n=1}^{\infty} \binom{m}{n} x^n, \quad |x| < 1$