APPM 1360 **Exam #3** **Fall 2009**

INSTRUCTIONS: Books, notes, and electronic devices are not permitted. On the front of your bluebook write: (1) **your name**, (2) **instructor's name**, and (3) "**TEST 3/FALL 2009**". Make a **scoring table** with room for 5 problems and a total score. **Work all problems. Start each problem on a new page. Clearly mark your answers.** A correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. **SHOW ALL WORK.**

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1. (16 pts) Do the following sequences converge or diverge? If convergent, give the limit. Explicitly show your reasoning.

   (a) $a_n = (-1)^n \dfrac{n}{n+1}$ (b) $\left\{(-1)^n \dfrac{n}{n^2+1}\right\}_{n\geq 22}$

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2. (20 pts) For each of the following series, determine whether the series converges absolutely, converges conditionally, or diverges. Justify your answers.

   (a) $\displaystyle\sum_{n=10}^{\infty} \frac{\cos(n\pi)}{n^{3/2}-1}$ (b) $\displaystyle\sum_{n=21}^{\infty} \frac{(-1)^n}{\ln(n)+n}$ (c) $\displaystyle\sum_{n=88}^{\infty} ne^{-n}$

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3. (20 pts) Show all work.

   (a) Use series to evaluate: $\displaystyle\lim_{x\to 0} \frac{\sin(x) - \tan^{-1}(x)}{x^3}$.

   (b) Find the **Taylor Polynomial of order 2** generated by $f(x) = (1-x)^{1/2}$ at $x = 0$.

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4. (24 pts) Justify your answers.

   (a) Find a Maclaurin Series for $f(x) = xe^{-x/2}$. (*You may use your knowledge of the Maclaurin Series of e^x to answer this question.*)

   (b) Use the first 3 non-zero terms of the series found in part (a) to approximate $\displaystyle\int_0^1 xe^{-x/2}\,dx$.

   (c) Estimate the error of the approximation found in part (b).

   (d) Is the approximation found in part (b) an *underestimate* or an *overestimate*?

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5. (20 pts) Show all work.

   (a) Find the interval of convergence of $\displaystyle\sum_{n=0}^{\infty}(n+1)x^n$.

   (b) For what values of x is the series conditionally convergent? absolutely convergent? divergent?

   (c) Show that the series in part (a) converges to $\dfrac{1}{(1-x)^2}$.

   (d) Find the sum of the series $\displaystyle\sum_{n=0}^{\infty}\frac{n+1}{3^n}$.

THERE ARE SOME USEFUL FORMULAS ON THE OTHER SIDE!