FORMULA SHEET

Some identities

$\cos(2x) = \cos^2(x) - \sin^2(x)$
$\sin(2x) = 2\sin(x)\cos(x)$
$\cos^2(x) = \dfrac{1+\cos(2x)}{2}$
$\sin^2(x) = \dfrac{1-\cos(2x)}{2}$

$\cosh^2(x) - \sinh^2(x) = 1$
$\cosh^2(x) = \dfrac{\cosh(2x)+1}{2}$
$\sinh^2(x) = \dfrac{\cosh(2x)-1}{2}$

Inverse Trigonometric Integral Identities

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}(\frac{u}{a}) + C,\ u^2 < a^2$$
$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}(\frac{u}{a}) + C$$
$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C,\ u^2 > a^2$$

Inverse Hyperbolic-Trig Integral Identities

$$\int \frac{du}{\sqrt{a^2+u^2}} = \sinh^{-1}(\frac{u}{a}) + C,\ a > 0$$
$$\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}(\frac{u}{a}) + C,\ u > a > 0$$
$$\int \frac{du}{a^2-u^2} = \frac{1}{a}\tanh^{-1}(\frac{u}{a}) + C,\quad \text{if } u^2 < a^2$$
$$\int \frac{du}{a^2-u^2} = \frac{1}{a}\coth^{-1}(\frac{u}{a}) + C,\quad \text{if } u^2 > a^2$$
$$\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\text{sech}^{-1}(\frac{u}{a}) + C,\ 0 < u < a$$
$$\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\text{csch}^{-1}\left|\frac{u}{a}\right| + C,\ u \neq 0$$

Moments, Mass and Center of Mass of a Thin Rod along x-axis

Mass: $M = \int_a^b \delta(x)dx$
Moments: $M_O = \int_a^b x\delta(x)dx$
Center of Mass: $\bar{x} = M_O/M$

Moments, Mass and Center of Mass of a Thin Plate

Mass: $M = \int dm,$
Moments: $M_x = \int \tilde{y}\ dm,\ M_y = \int \tilde{x}\ dm$
Center of Mass: $\bar{x} = M_y/M,\ \bar{y} = M_x/M$

Volume of a Solid of Revolution

Disk Method: $\Delta V = \pi R^2 \Delta(\cdot)$
Washer Method: $\Delta V = \pi[R^2 - r^2]\Delta(\cdot)$
Shell Method: $\Delta V = 2\pi r h \Delta(\cdot)$

Differential form Arclength/Surface Area

Given the arclength differential $ds = \sqrt{dx^2 + dy^2}$, then arclength is given by

$$L = \int ds$$

and the differential formula of the *surface area* of a curve rotated about the x-axis is $SA = \int 2\pi y ds$.

Some useful limits

$$\lim_{n\to\infty} \frac{\ln n}{n} = 0 \qquad \lim_{n\to\infty} \sqrt[n]{n} = 1$$

$$\lim_{n\to\infty} x^{1/n} = 1,\ x > 0 \qquad \lim_{n\to\infty} x^n = 0,\ |x| < 1$$
$$\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = e^x, \text{ any } x \qquad \lim_{n\to\infty} \frac{x^n}{n!} = 0, \text{ any } x$$