FORMULA SHEET

Some identities

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$
$$\sin(2x) = 2\sin(x)\cos(x)$$
$$\cos^2(x) = \frac{1+\cos(2x)}{2}$$
$$\sin^2(x) = \frac{1-\cos(2x)}{2}$$

$$\cosh^2(x) - \sinh^2(x) = 1$$
$$\cosh^2(x) = \frac{\cosh(2x)+1}{2}$$
$$\sinh^2(x) = \frac{\cosh(2x)-1}{2}$$

Inverse Trigonometric Integral Identities

$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}(\frac{u}{a}) + C,\ u^2 < a^2$$
$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}(\frac{u}{a}) + C$$
$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C,\ u^2 > a^2$$

Inverse Hyperbolic-Trig Integral Identities

$$\int \frac{du}{\sqrt{a^2+u^2}} = \sinh^{-1}(\frac{u}{a}) + C,\ a > 0$$
$$\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}(\frac{u}{a}) + C,\ u > a > 0$$
$$\int \frac{du}{a^2-u^2} = \frac{1}{a}\tanh^{-1}(\frac{u}{a}) + C,\quad \text{if } u^2 < a^2$$
$$\int \frac{du}{a^2-u^2} = \frac{1}{a}\coth^{-1}(\frac{u}{a}) + C,\quad \text{if } u^2 > a^2$$
$$\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\text{sech}^{-1}(\frac{u}{a}) + C,\ 0 < u < a$$
$$\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\text{csch}^{-1}\left|\frac{u}{a}\right| + C,\ u \neq 0$$

Frequently used Maclaurin series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad |x| < 1$$
$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad |x| < \infty$$
$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \quad |x| < \infty$$
$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \quad |x| < \infty$$
$$\tan^{-1}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}, \quad |x| \le 1$$
$$(1+x)^m = 1 + \sum_{k=1}^{\infty} \binom{m}{k} x^k, \quad |x| < 1$$

Some useful limits

$$\lim_{n\to\infty} \frac{\ln n}{n} = 0 \qquad \lim_{n\to\infty} \sqrt[n]{n} = 1$$

$$\lim_{n\to\infty} x^{1/n} = 1,\ x > 0 \qquad \lim_{n\to\infty} x^n = 0,\ |x| < 1$$
$$\lim_{n\to\infty} \left(1+\frac{x}{n}\right)^n = e^x, \text{ any } x \qquad \lim_{n\to\infty} \frac{x^n}{n!} = 0, \text{ any } x$$