

## FORMULA SHEET

### Some identities

$$\begin{aligned}\cos^2(x) + \sin^2(x) &= 1 \\ \cos(2x) &= \cos^2(x) - \sin^2(x) \\ \sin(2x) &= 2 \sin(x) \cos(x) \\ \cos^2(x) &= \frac{1 + \cos(2x)}{2} \\ \sin^2(x) &= \frac{1 - \cos(2x)}{2}\end{aligned}$$

$$\begin{aligned}\cosh^2(x) - \sinh^2(x) &= 1 \\ \cosh^2(x) &= \frac{\cosh(2x) + 1}{2} \\ \sinh^2(x) &= \frac{\cosh(2x) - 1}{2}\end{aligned}$$

### Inverse Trigonometric Integral Identities

$$\begin{aligned}\int \frac{du}{\sqrt{a^2 - u^2}} &= \sin^{-1}\left(\frac{u}{a}\right) + C, \quad u^2 < a^2 \\ \int \frac{du}{a^2 + u^2} &= \frac{1}{a} \tan^{-1}\left(\frac{u}{a}\right) + C \\ \int \frac{du}{u\sqrt{u^2 - a^2}} &= \frac{1}{a} \sec^{-1}\left|\frac{u}{a}\right| + C, \quad u^2 > a^2\end{aligned}$$

### Inverse Hyperbolic-Trig Integral Identities

$$\begin{aligned}\int \frac{du}{\sqrt{a^2 + u^2}} &= \sinh^{-1}\left(\frac{u}{a}\right) + C, \quad a > 0 \\ \int \frac{du}{\sqrt{u^2 - a^2}} &= \cosh^{-1}\left(\frac{u}{a}\right) + C, \quad u > a > 0 \\ \int \frac{du}{a^2 - u^2} &= \frac{1}{a} \tanh^{-1}\left(\frac{u}{a}\right) + C, \quad \text{if } u^2 < a^2 \\ \int \frac{du}{a^2 + u^2} &= \frac{1}{a} \coth^{-1}\left(\frac{u}{a}\right) + C, \quad \text{if } u^2 > a^2 \\ \int \frac{du}{u\sqrt{a^2 - u^2}} &= -\frac{1}{a} \operatorname{sech}^{-1}\left(\frac{u}{a}\right) + C, \quad 0 < u < a \\ \int \frac{du}{u\sqrt{a^2 + u^2}} &= -\frac{1}{a} \operatorname{csch}^{-1}\left|\frac{u}{a}\right| + C, \quad u \neq 0\end{aligned}$$

### Arclength of curve

If the curve  $x = f(t)$ ,  $y = g(t)$ ,  $a \leq t \leq b$  is traversed exactly once as  $t$  increases from  $a$  to  $b$ , then the curve's length is

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \int_a^b ds$$

### Moments, Mass and Center of Mass of a Thin Wire (2D)

$$\text{Mass: } M = \int dm = \int \delta ds$$

$$\text{Moments: } M_x = \int \tilde{y} dm, \quad M_y = \int \tilde{x} dm$$

$$\text{Center of Mass: } \bar{x} = M_y/M, \quad \bar{y} = M_x/M$$

## Volume of a Solid of Revolution

$$\text{Disk Method: } \Delta V = \pi R^2 \Delta(\cdot)$$

$$\text{Washer Method: } \Delta V = \pi[R^2 - r^2] \Delta(\cdot)$$

$$\text{Shell Method: } \Delta V = 2\pi rh \Delta(\cdot)$$

### Some useful limits

$$\lim_{n \rightarrow \infty} \frac{\ln n}{n} = 0$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{n} = 1$$

$$\lim_{n \rightarrow \infty} x^{1/n} = 1, \quad x > 0$$

$$\lim_{n \rightarrow \infty} x^n = 0, \quad |x| < 1$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n = e^x, \quad \text{any } x \quad \lim_{n \rightarrow \infty} \frac{x^n}{n!} = 0, \quad \text{any } x$$

### Frequently used Maclaurin series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad |x| < 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad |x| < \infty$$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \quad |x| < \infty$$

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \quad |x| < \infty$$

$$\tan^{-1}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}, \quad |x| \leq 1$$

$$(1+x)^m = 1 + \sum_{k=1}^{\infty} \binom{m}{k} x^k, \quad |x| < 1$$

### Ellipses

For the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ,  $a > b$ , we have

$$\text{Center-to-focus distance: } c = \sqrt{a^2 - b^2}$$

$$\text{Foci: } (\pm c, 0)$$

$$\text{Vertices: } (\pm a, 0)$$

$$\text{Eccentricity: } e = c/a, \quad \text{Directrix: } x = \pm a/e$$

### Hyperbolas

For the hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ , we have

$$\text{Center-to-focus distance: } c = \sqrt{a^2 + b^2}$$

$$\text{Foci: } (\pm c, 0)$$

$$\text{Vertices: } (\pm a, 0)$$

$$\text{Asymptotes: } y = \pm \frac{b}{a}x$$

$$\text{Eccentricity: } e = c/a, \quad \text{Directrix: } x = \pm a/e$$