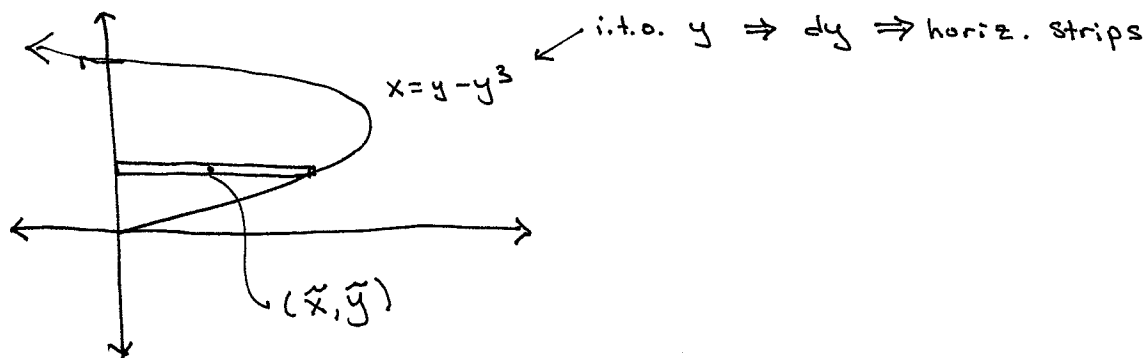


5.7 continued... examples

$\delta$  constant

# 17      y-axis and       $x = y - y^3$        $0 \leq y \leq 1$



$$l = y - y^3$$

$$w = dy$$

$$dA = l \cdot w = (y - y^3) dy$$

$$\text{mass} = dm = \delta dA = \delta (y - y^3) dy$$

$$\tilde{x} = \frac{y - y^3}{2}$$

$$\tilde{y} = y$$

$$\bar{y} = \frac{M_x}{M} = \frac{\int_0^1 \tilde{y} dm}{\int_0^1 dm} = \frac{\int_0^1 y \cdot \delta (y - y^3) dy}{\int_0^1 \delta (y - y^3) dy} = \frac{2\delta/15}{\delta/4} = \frac{8}{15}$$

$$\bar{x} = \frac{M_y}{M} = \frac{\int_0^1 \tilde{x} dm}{\int_0^1 dm} = \frac{\int_0^1 \frac{(y - y^3)}{2} \cdot \delta (y - y^3) dy}{\int_0^1 \delta (y - y^3) dy} = \frac{4\delta/105}{\delta/4} = \frac{16}{105}$$

$$(\bar{x}, \bar{y}) = \left( \frac{16}{105}, \frac{8}{15} \right)$$

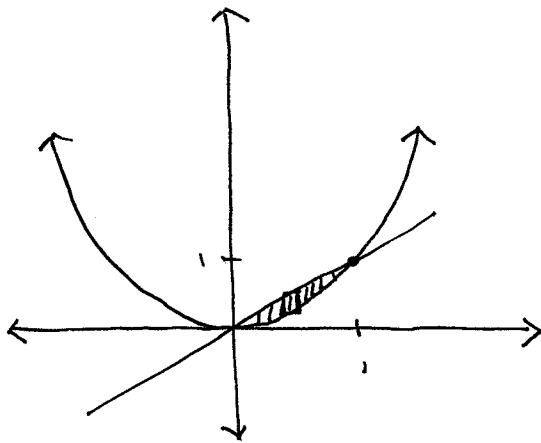
\* "Centroid"  $\Rightarrow$  Constant Density

\* Use symmetry!

$\delta$  - varying

#26 Bounded below by  $y = x^2$   
Bounded above by  $y = x$

$$\delta(x) = 12x$$



vertical strip

$$l = x - x^2$$

$$dA = l \cdot w = (x - x^2) dx$$

$$w = dx$$

$$\text{mass} = dm = \delta dA = 12x(x - x^2) dx$$

$$\tilde{x} = x$$

$$\tilde{y} = \frac{x + x^2}{2}$$

$$\bar{y} = \frac{M_x}{M} = \frac{\int_0^1 \tilde{y} dm}{\int_0^1 dm} = \frac{\int_0^1 \frac{x + x^2}{2} \cdot 12x(x - x^2) dx}{\int_0^1 12x(x - x^2) dx} = \frac{1/2}{1}$$

$$\bar{x} = \frac{M_y}{M} = \frac{\int_0^1 \tilde{x} dm}{\int_0^1 dm} = \frac{\int_0^1 x \cdot 12x(x - x^2) dx}{\int_0^1 12x(x - x^2) dx} = \frac{3/5}{1}$$

$$(\bar{x}, \bar{y}) = \left(\frac{3}{5}, \frac{1}{2}\right)$$