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Lecture
Friday
9-18-78

Ch. 7.1

Review of Integration Techniques

1. Substitution – Put into managable form
2. Complete the Square
3. Trig. Identity
4. Eliminate a Square Root
5. Reducing Improper Fractions
6. Multiply by a form of 1.

1. Ex:

$$\int \frac{dx}{x-\sqrt{x}}$$

$$u = \sqrt{x} - 1$$

$$du = \frac{1}{2\sqrt{x}}\, dx$$

$$2du = \frac{dx}{\sqrt{x}}$$

$$= \int \frac{dx}{\sqrt{x}(\sqrt{x}-1)}$$

$$= \int \frac{2\,du}{u}$$

$$= 2\ln|u| + c$$

$$= 2\ln|\sqrt{x}-1| + c$$

$1\frac{1}{2}$ **Ex:** $\displaystyle\int_{\frac{\pi}{2}}^{\pi} \sin y \; e^{\cos y} \, dy$

$u = \cos y$
$du = -\sin y \, dy$
$-du = \sin y \, dy$

$$= -\int_{0}^{-1} e^{u} \, du$$

$$= \int_{-1}^{0} e^{u} \, du$$

$$= e^{u} \Big|_{-1}^{0}$$

$$= e^{0} - e^{-1}$$

$$\boxed{= 1 - \frac{1}{e}}$$

2. **Ex:** $\displaystyle\int \frac{dx}{(x+1)\sqrt{x^2+2x}}$

complete the square under the radical

$$= \int \frac{dx}{(x+1)\sqrt{x^2+2x+1-1}}$$

$$= \int \frac{dx}{(x+1)\sqrt{(x^{\cancel{2}}+1)^2-1}}$$

$u = x+1$
$du = dx$

$$= \int \frac{du}{u\sqrt{u^2-1}} = \sec^{-1}|u| + c$$

$$\boxed{= \sec^{-1}|x+1| + c}$$

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3. Ex: $\int (\csc x - \tan x)^2 \, dx$

$$= \int \csc^2 x - 2\csc x \tan x + \tan^2 x$$

$$\int \frac{1-x}{\sqrt{1-x^2}}\,dx$$

$$= \int\left(\frac{1}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}}\right)dx$$

$$= \int \frac{dx}{\sqrt{1-x^2}} - \int \frac{x}{\sqrt{1-x^2}}\,dx$$

$u = 1-x^2$

$du = -2x\,dx$

$\frac{du}{2} = -x\,dx$

$$= \sin^{-1} x$$

$$+ \int \frac{du}{2\sqrt{u}}$$

$$+ \frac{1}{2}\int u^{-1/2}\,du$$

$$+ \frac{1}{2}\left(\frac{u^{1/2}}{1/2}\right)$$

$$+ \sqrt{u}$$

$$+ \sqrt{1-x^2}$$

$$= \sin^{-1} x + \sqrt{1-x^2} + c$$

∎

Ex: $\int_0^{\pi} \sqrt{1-\cos 2x}\ dx$

$\sin^2 x = \frac{1-\cos 2x}{2}$

$2\sin^2 x = 1-\cos 2x$

$= \int_0^{\pi} \sqrt{2\sin^2 x}\ dx$

$= \sqrt{2}\int_0^{\pi} \sin x\ dx$

$= -\sqrt{2}\cos x \Big|_0^{\pi}$

$= -\sqrt{2}\left[\cos\pi - \cos 0\right]$

$= -\sqrt{2}\left[-1-1\right]$

$= \boxed{2\sqrt{2}}$

$$\frac{\sin^2}{\cos^2} + \frac{\cos^2}{\cos^2} = \frac{1}{\cos^2}$$

$$\tan^2 + 1 = \sec^2$$
$$1 = \sec^2 - \tan^2$$

Ex: $\displaystyle\int \frac{d\theta}{\sec\theta + \tan\theta}$

$$= \int \frac{d\theta}{\sec\theta + \tan\theta} \cdot \frac{\sec\theta - \tan\theta}{\sec\theta - \tan\theta}$$

$$= \int \frac{\sec\theta - \tan\theta}{\sec^2\theta - \tan^2\theta} \, d\theta$$

$$= \int \sec\theta - \tan\theta \quad d\theta$$

$$= \int \sec\theta - \int \tan\theta$$

$$= \int \sec\theta \cdot \frac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta} - \int \frac{\sin\theta}{\cos\theta} \, d\theta$$

$u = \cos\theta$

$du = -\sin\theta \, d\theta$

$u = \sec\theta + \tan\theta$

$du = \sec\theta \tan\theta + \sec^2\theta$

$$+ \int \frac{1}{u} \, du$$

$$= \int \frac{(\sec^2\theta + \sec\theta\tan\theta)d\theta}{\sec\theta + \tan\theta}$$

$$+ \ln|\cos\theta|$$

$$= \int \frac{du}{u}$$

$$= \ln|\sec\theta + \tan\theta| + \ln|\cos\theta| + C$$