

Exam 1 Review 6.10, 6.11, 5.1-5.7

Test Topics

1. Hyperbolic Functions

- * def
- * derivs
- * integrals

2. First Order ODE's

- * Seperable
- * First Order Linear

3. Areas between two curves

4. Volumes by Slicing

5. Disk / Washer Method for volume of Solid of ~~Rotation~~ Revolution

6. Shell Method " " " " "

7. Lengths of Plane Curves

8. Surface Area for solids of Revolution

9. Moments and Centers of Mass

Hyp. Functions

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$(\cosh x)' = \sinh x$$

$$(\sinh x)' = \cosh x$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

etc.

Inv. Hyp. Trig. Functions: useful for integration

$$1. \int \frac{du}{\sqrt{a^2 + u^2}} = \sinh^{-1} \left(\frac{u}{a} \right) + C$$

$$2. \int \frac{du}{\sqrt{u^2 - a^2}} = \cosh^{-1} \left(\frac{u}{a} \right) + C$$

$$3. \int \frac{du}{a^2 - u^2} = \begin{cases} \frac{1}{a} \tanh^{-1} \left(\frac{u}{a} \right) + C & u^2 < a^2 \\ \frac{1}{a} \coth^{-1} \left(\frac{u}{a} \right) + C & a^2 < u^2 \end{cases}$$

$$4. \int \frac{du}{u\sqrt{a^2 - u^2}} = -\frac{1}{a} \operatorname{sech}^{-1} \left(\frac{u}{a} \right) + C$$

$$5. \int \frac{du}{u\sqrt{a^2 + u^2}} = -\frac{1}{a} \operatorname{csch}^{-1} \left| \frac{u}{a} \right| + C$$

Ex: $\int_1^2 \frac{dx}{x\sqrt{4+x^2}} = -\frac{1}{2} \operatorname{csch}^{-1} \left| \frac{x}{2} \right| \Big|_1^2 = -\frac{1}{2} \left[\operatorname{csch}^{-1}(1) - \operatorname{csch}^{-1}\left(\frac{1}{2}\right) \right]$



First Order ODE

* Separable - Try to sep. first.
If not poss, put in form...

* First Order Linear

$$\frac{dy}{dx} + P(x)y = Q(x)$$

Then solution is

$$y = \frac{1}{v(x)} \int v(x)Q(x) dx \quad \text{where } v(x) = e^{\int P(x)dx}$$

Ex:

~~sin x cos x~~

$$\frac{y'}{\sin x} + \frac{y}{\cos x} = \frac{\cos^2 x}{\sin x} \cot x \cdot \cos x$$

Soln: $y = \sin x \cos x + C \cos x$