1360
Lecture
Friday
9-25-09

7.2 continued
7.3

7.2 Integration by Parts

Basic Formula

Indefinite

$$\int u\,dv = uv - \int v\,du$$

Definite

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

How to choose u and dv

u - easy to differentiate

dv - easy to integrate

When to use

(polynomial)(trig)

(polynomial)(exponential)

(trig)(exponential)

(ln)(polynomial)

* Special Cases - u-sub beforehand.

Ann's personal favorite

$$\int e^x \cos x \, dx \quad \text{and} \quad \int e^x \sin x \, dx$$

$$\int e^x \sin x \, dx$$

$u = e^x$ $\qquad$ $v = -\cos x$

$du = e^x dx$ $\qquad$ $dv = \sin x dx$

$$\int e^x \sin x dx = -e^x \cos x - \int (-\cos x) e^x \, dx$$

$$\int e^x \sin x dx = -e^x \cos x + \int e^x \cos x \, dx$$

$u = e^x$ $\qquad$ $v = \sin x$

$du = e^x dx$ $\qquad$ $dv = \cos x dx$

$$\int e^x \sin x dx = -e^x \cos x + e^x \sin x - \int \sin x \cdot e^x dx$$

add to both sides

$$2\int e^x \sin x \, dx = -e^x \cos x + e^x \sin x$$

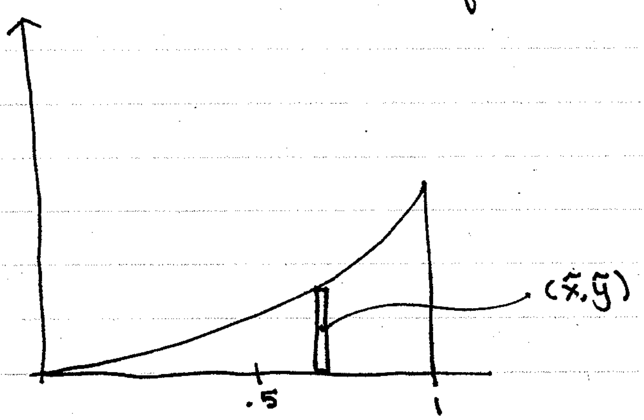
$$\int e^x \sin x dx = \frac{e^x}{2}\left[\sin x - \cos x\right]$$

Ex: # 37

Find Centroid of a Thin Plate covering the region bounded by

$$y = x^2 e^x, \quad x\text{-axis} \quad \text{and} \quad x = 1$$

in the first quadrant.



$\ell = x^2 e^x$ $\qquad$ $dA = \ell \cdot w = x^2 e^x dx$

$w = dx$ $\qquad$ $dm = \delta\, dA = \delta \cdot x^2 e^x dx$

$\tilde{x} = x$

$\tilde{y} = \dfrac{x^2 e^x}{2}$

$$M_y = \int_0^1 \tilde{x}\, dm = \int_0^1 x \cdot \delta x^2 e^x \, dx =$$

$$M_x = \int_0^1 \tilde{y}\, dm = \int_0^1 \frac{x^2 e^x}{2} \cdot \delta x^2 e^x \, dx =$$

$$M = \int_0^1 dm = \int_0^1 \delta x^2 e^x \, dx =$$

$$(\bar{x}, \bar{y}) = \left(\frac{6 - 2e}{e - 2}, \frac{e^2 - 3}{e - 2} \right)$$

7.3 Partial Fractions

* Undoing finding a common denominator for Rational Functions

$f(x) = \dfrac{P(x)}{Q(x)}$ $\qquad P(x), Q(x)$ polynomials

* deg. of $P(x)$ must be less than deg. $Q(x)$

Ex:

#17 $\displaystyle\int_0^1 \frac{x^3}{x^2+2x+1}\,dx = 3\ln 2 - 2$

$\deg(P) > \deg(Q) \Rightarrow$ polynomial long division

$$
\begin{array}{r|l}
 & x - 2 \\
x^2+2x+1 & x^3 + 0x^2 + 0x + 0 \\
 & \underline{x^3 + 2x^2 + x} \\
 & -2x^2 - x + 0 \\
 & \underline{-2x^2 - 4x - 2} \\
 & 3x + 2
\end{array}
$$

↑ Remainder

$$= x - 2 + \frac{3x+2}{x^2+2x+1}$$

Consider $\dfrac{3x+2}{(x+1)(x+1)} = \dfrac{A}{x+1} + \dfrac{B}{(x+1)^2}$

$$3x+2 = A(x+1)^2 + B(x+1)$$

solve for A, B

Easy Cases: No repeated factors

Ex: Distinct Linear Factors

$$\frac{z}{z^3 - z^2 - 6z}$$

Ex: Distinct Linear & Higher Order Factors

$$\frac{1}{(x+1)(x^2+3)}$$