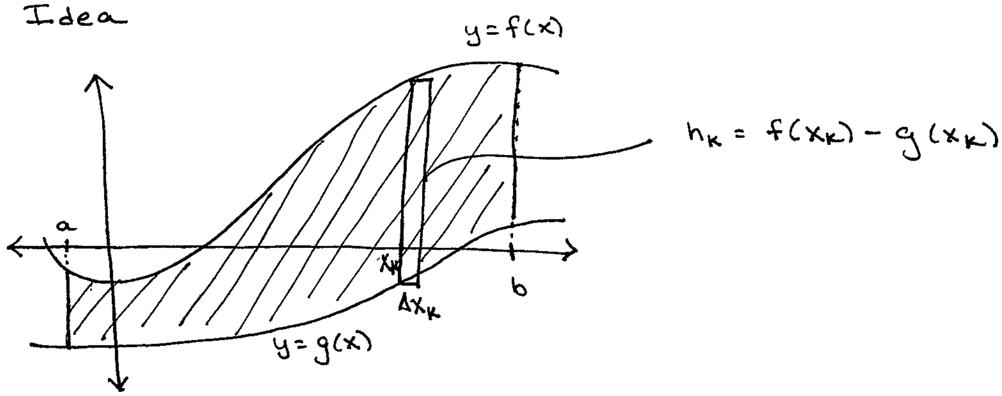
1360
Lecture
Monday
8-31-09

5.1 Areas Between Curves

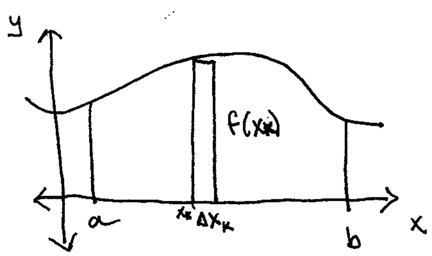
* Straightforward Concept
* Details can get tricky

General Idea



Given two curves $y=f(x)$, $y=g(x)$, suppose we want the area between the two curves

Recall, area under curve & above x-axis, is Riemann sum... infinitesimally thin rectangles



$A = w \cdot h$

$\Delta A_K = \Delta x_K \cdot f(x_K)$

$$A = \lim_{\|P\| \to 0} \sum_{K=1}^{n} \Delta A_K = \lim_{\|P\| \to 0} \sum_{K=1}^{n} \Delta x_K f(x_K) = \int_a^b f(x)\, dx$$

Now, with two curves

* recall from theory... $\|P\|$ not necess. uniform & can evaluate at any pt in int Δx_K.

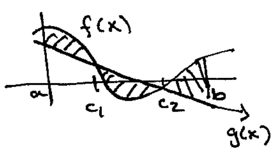
Since height of Rectangles is only changes we immediately get

Area Btwn Curves is

$$\lim_{\|P\| \to 0} \sum_{k=1}^{n} \Delta x_k \left[f(x_k) - g(x_k) \right]$$

$$= \int_a^b (f(x) - g(x))\, dx$$

In Practice... details can be the tricky part

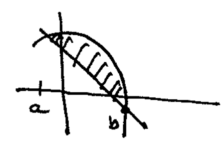


$\int_a^{c_1} f-g + \int_{c_1}^{c_2} g-f + \int_{c_2}^{b} f-g$

* if curves intersect, upper & lower may switch on $[a, b]$

* '' '' '' Need to find int. ~~pt~~ pt. to know int. limits

* & of course, integration!



rest is infinite.

Steps to solve

1. Graph!!!
2. Find integration limits
3. Simplify $f(x) - g(x)$ as much as possible
4. Integrate.

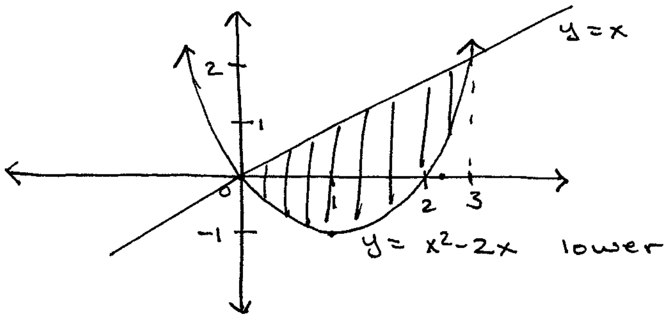
* In simplification - don't forget trig identities

* If there is symmetry... use it!

$y = x^2 - 2x + 1 - 1$ $\qquad -1$

$y = (x-1)^2 - 1$ $\qquad 1$

Ex: $\quad y = x^2 - 2x \quad$ and $\quad y = x$



$x^2 - 2x = x$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0, 3$

$$\int_0^3 [x - (x^2 - 2x)]\, dx$$

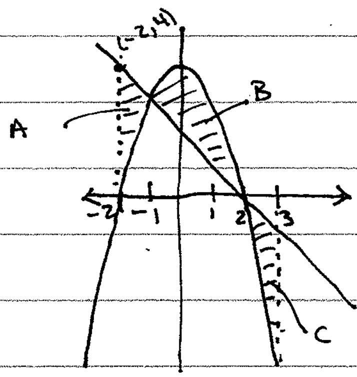
$$= \int_0^3 -x^2 + 3x \quad dx$$

$$= \left. \frac{-x^3}{3} + \frac{3x^2}{2} \right|_0^3 = \left[-\frac{27}{3} + \frac{27}{2}\right] - \cancel{[-0 + 0]} \to 0$$

$$= -9 + \frac{27}{2} = -\frac{18}{2} + \frac{27}{2} = \boxed{\frac{9}{2}}$$

11

Ex: $y = 4 - x^2$ $y = -x + 2$



$$A = \int_{-2}^{-1} (-x+2) - (4-x^2) = \int_{-2}^{1} (x^2 - x - 2)\,dx = \boxed{\frac{11}{6}} \; A$$

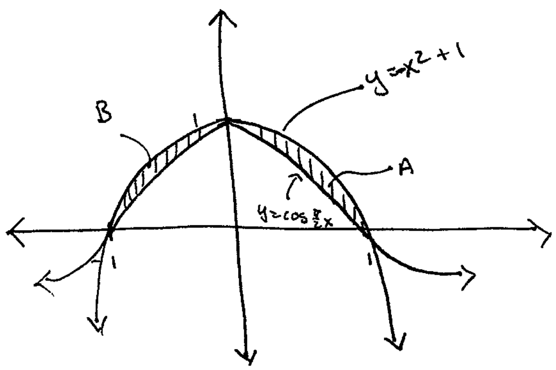
$$B = \int_{-1}^{2} (4-x^2) - (-x+2) = \int_{-1}^{2} -x^2 + x + 2 = \boxed{\frac{9}{2}} \; B$$

$$C = \int_{2}^{3} (-x+2) - (4-x^2) = \int_{2}^{3} x^2 - x - 2 = \boxed{9 - \frac{9}{2} - \frac{8}{3}} \; C$$

$$A + B + C = \boxed{\frac{49}{6}} \text{ total}$$

Ex: #38 5.1

$y = \cos(\frac{\pi}{2}x)$ and $y = 1 - x^2$



Symmetric. Find A, mult by 2

$$A = \int_0^1 (1-x^2) - (\cos\tfrac{\pi}{2}x) \quad dx$$

$$= x - \frac{x^3}{3} - \frac{\sin\frac{\pi}{2}x}{\frac{\pi}{2}} \Bigg|_0^1$$

$$= x - \frac{x^3}{3} - \frac{2}{\pi}\cdot\sin\tfrac{\pi}{2}x \Bigg|_0^1$$

$$= \left(1 - \frac{1}{3} - \frac{2}{\pi}\overset{1}{\cancel{\sin\tfrac{\pi}{2}}}\right) - \left(0 - 0 - \frac{2}{\pi}\overset{0}{\cancel{\sin 0}}\right)$$

$$= \frac{2}{3} - \frac{2}{\pi}$$

Mult by 2

$$\boxed{\frac{4}{3} - \frac{4}{\pi}}$$

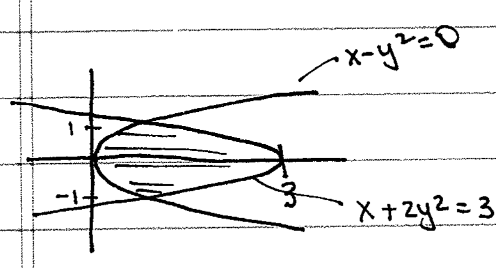
correct by back of Book!!

* if not sure which is upper & lower fn, might have done ☺ $\cos\frac{\pi}{2}x - (1-x^2)$ ⇝ get $\frac{4}{\pi} - \frac{4}{3}$

* check: is $\boxed{\text{neg}}$

#26

Ex: $x - y^2 = 0$ and $x + 2y^2 = 3$



$$A = \int_{-1}^{1} (3-2y^2) - y^2 = \int_{-1}^{1} (3-3y^2)\,dy = \boxed{4}$$