1360
Lecture
Friday
9-4-09

Recall:

Given a region in xy-plane, the volume of the solid generated by revolving about the x-axis or any line // to x-axis, where $a \le x \le b$,

$$V = \int_a^b \pi [R(x)]^2 \, dx$$

$R[x]$ = radius of a circular Cross-Section i.t.o. x

Similarly, given a region in xy-plane, volume of solid gen. by rev. about y-axis or // to y-axis, where $c \le y \le d$ is given by

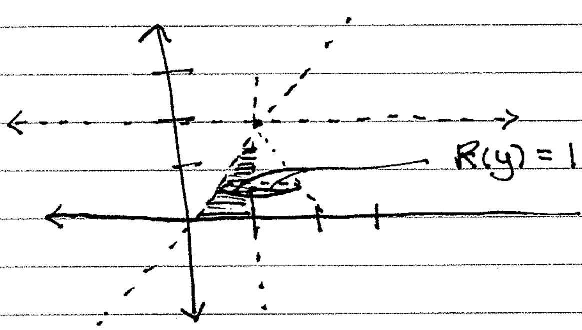
$$V = \int_c^d \pi [R(y)]^2 \, dy$$

$R(y)$ = radius of circular Cross-Section i.t.o. y

$\boxed{\text{Disk Method}}$

W

<u>Ex:</u> Find volume of solid gen. by revolving the region bounded by $y = 2x$, $y = 0$ and $x = 1$ about the line $x = 1$



// to y-axis so dy

$R(y) = 1 - x = 1 - \frac{y}{2}$

①

$$\int_{y=0}^{y=2} \pi \left(1-\frac{y}{2}\right)^2 dy$$

$$= \pi \int_0^2 \frac{y^2}{4} - y + 1 \; dy$$

$$= \pi \left[\frac{y^3}{12} - \frac{y^2}{2} + y \right]_0^2$$

$$= \pi \left[\frac{8}{12} - \frac{4}{2} + 2 \right]$$

$$= \frac{2}{3}\pi$$

■

Washer Method (Previewed yesterday in Lecture)

Extend Disc Method to disc w/ hole - Sub. area of hole from Area of entire disc cross-cross section.

Revolve about x-axis or // to it

$$\int_a^b \pi \left[[R(x)]^2 - [r(x)]^2 \right] dx$$

$R(x)$ = Outer Radius , $r(x)$ = Inner Radius

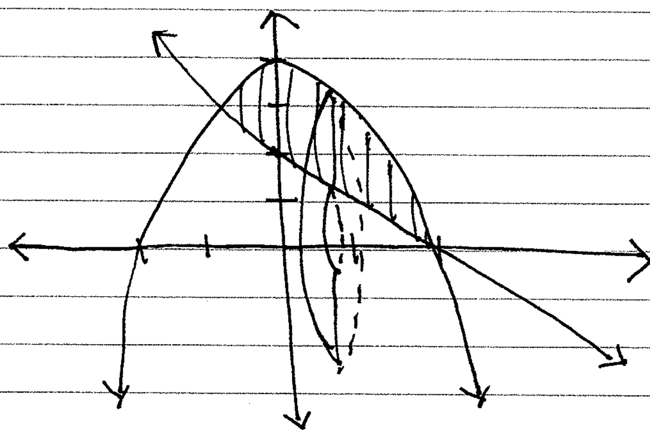
Revolve about y-axis or // to it

$$\int_c^d \pi \left[R(y)^2 - r(y)^2 \right] dy$$

Ex: Consider the region bounded by curves

#26 $y = 4 - x^2$ and $y = 2 - x$

Revolve about x-axis



$R(x) = 4 - x^2$

$r(x) = 2 - x$

$$\int_0^2 \pi \left[(4-x^2)^2 - (2-x)^2 \right] dx$$

$$= \pi \int_0^2 (x^4 - 9x^2 + 4x + 12) \, dx$$

$$= \pi \left[\frac{x^5}{5} - 3x^3 + 2x^2 + 12x \right]_0^2$$

$$= \pi \left[\frac{32}{5} - \cancel{24} + 8 + \cancel{24} \right]$$

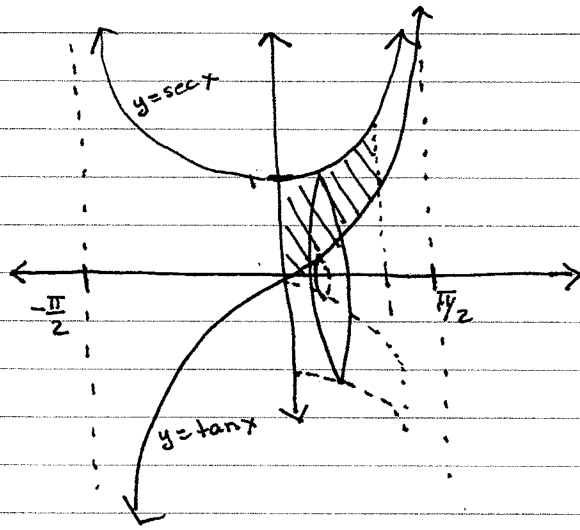
$$= \boxed{\frac{72\pi}{5}}$$

Consider the region bounded by

Ex: $y = \sec x$, ~~and~~ $y = \tan x$, $x = 0$ & $x = 1$

#28

Revolve about x-axis, find volume



$R(x) = \sec x \qquad r(x) = \tan x$

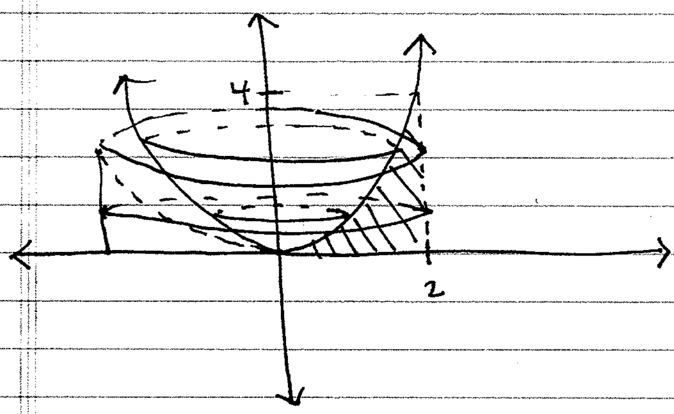
$$\int_0^1 \pi \left(\sec^2 x - \tan^2 x\right) dx$$

$$= \int_0^1 \pi \cdot 1 \, dx$$

$$= \pi x \Big|_0^1 = \boxed{\pi}$$

Ex: Consider the region ~~[illegible]~~ in the 1st Quadrant bounded by

$y = x^2$, x-axis & $x = 2$



revolve about y-axis
$\Rightarrow$ dy

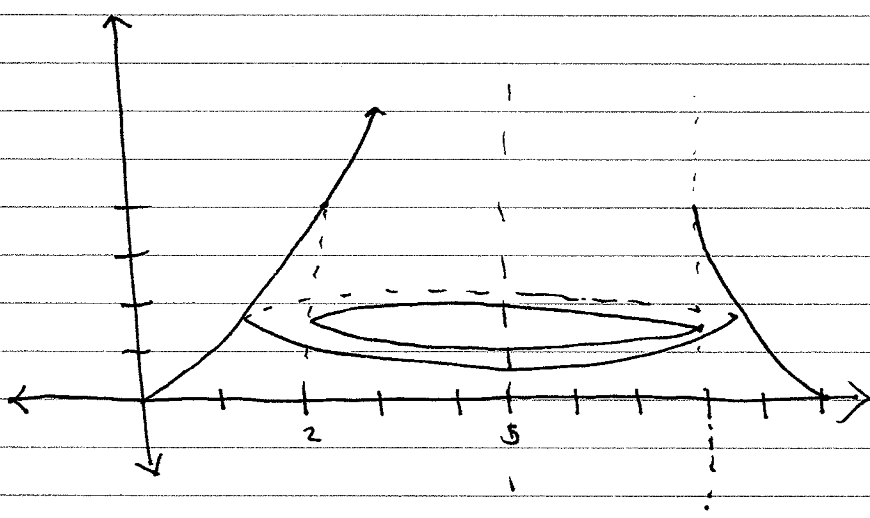
$R(y) = 2$
$r(y) = \sqrt{y}$

$$\int_{y=0}^{y=4} \pi\left(2^2 - \sqrt{y}^{\,2}\right) dy$$

$$= \pi\int_0^4 4 - y \;dy = \pi\left[4y - \frac{y^2}{2}\right]_0^4$$

$$= \pi\left[16 - 8\right] = \boxed{8\pi}$$

Now... revolve the same region about the line $x = 5$



Still dy

$$R(y) = 5 - \sqrt{y}$$

$$r(y) = 3$$

$$\int_0^4 \pi\left[(5-\sqrt{y})^2 - 3^2\right] dy$$

$$= \pi \int_0^4 (25 - 10\sqrt{y} + y - 9)\, dy$$

$$= \pi \int_0^4 16 - 10\sqrt{y} - y$$

$$= \pi\left[16y - \frac{20}{3}y^{3/2} - \frac{y^2}{2}\right]_0^4$$

$$= \pi\left[64 - \frac{160}{3} - 8\right]$$

$$= \boxed{\frac{8\pi}{3}}$$