1360
Lecture
Wednesday
9-9-09

5.4 Shell Method: Volumes of Solids of Revolution

Formulas

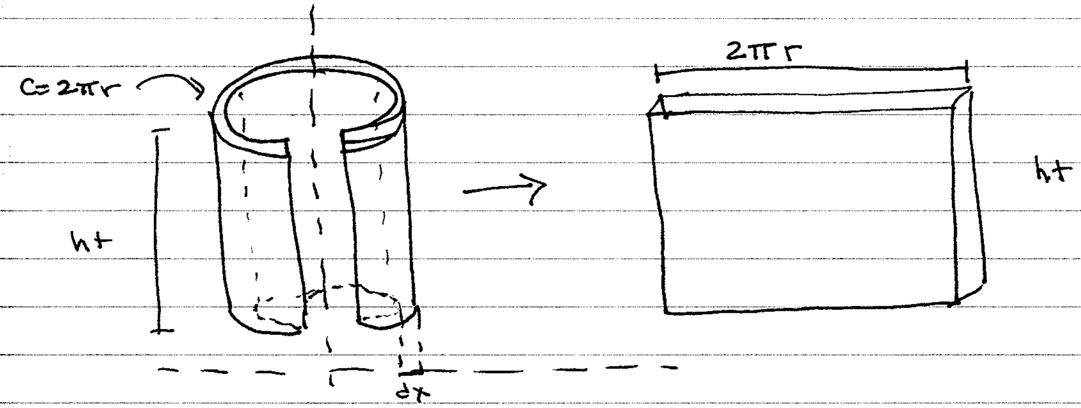
Rotate about y-axis $\Rightarrow$ dx
Rotate about x-axis $\Rightarrow$ dy

$\Big\}$ opp. from

Shell formula - Rev. about y-axis

Given a region btwn x-axis, & graph of $f(x) \geq 0$, with $0 \leq a \leq x \leq b$, volume ~~when rotate about~~ of solid formed when rotate about y-axis is

$$V = \int_a^b \underbrace{2\pi \begin{pmatrix}\text{shell}\\ \text{radius}\end{pmatrix}}_{\text{Circumf.}} \underbrace{\begin{pmatrix}\text{shell}\\ \text{height}\end{pmatrix}}_{\text{ht}} \underbrace{dx}_{\text{thickness}} = \int_a^b 2\pi x f(x)\,dx$$



$2\pi r h =$ Area.

infinitesimally thin "thickness" $= dx$

$\Rightarrow$ $2\pi r h\,dx$ = small volume

$\sum$ of infinitely many infinitely thin volumes $= \int_a^b$

$\Rightarrow$ Volume of Solid.

Shell formula – Rev. about x-axis ... same but dy

Given a region btwn y-axis & graph of $f(y) \geq 0$, where $0 \leq c \leq y \leq d$, the volume of solid gen. by revolving about the x-axis is

$$V = \int_c^d 2\pi \begin{pmatrix}\text{shell}\\ \text{radius}\end{pmatrix}\begin{pmatrix}\text{shell}\\ \text{ht}\end{pmatrix} dx = \int_c^d 2\pi y\, f(y)\, dy$$

Same as before, only now, the infinitesimally thin "thikness" is along y-axis $\Rightarrow dy$

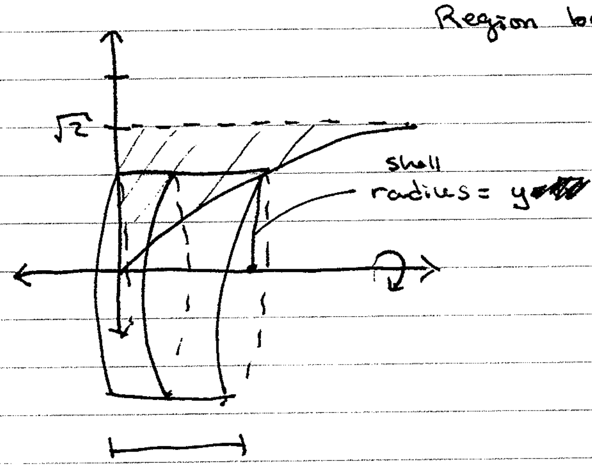
3

Ex:

Region bounded by $x = y^2$, y-axis and line $y = \sqrt{2}$

Revolve about x-axis

$\Rightarrow dy$



shell ht = y^2

she

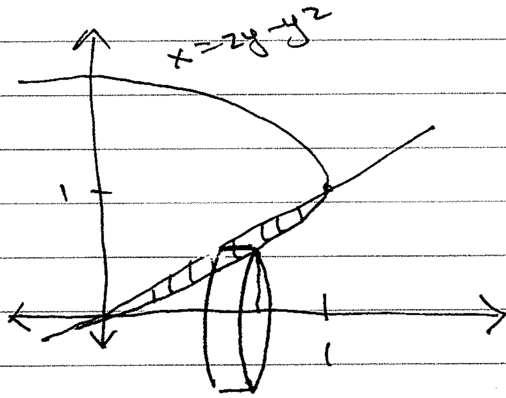
limits of integration $0 \le y \le \sqrt{2}$

$$V = \int_0^{\sqrt{2}} 2\pi \underbrace{y}_{\text{radius}} \cdot \underbrace{y^2}_{\text{ht}} \, dy$$

$$= 2\pi \int_0^{\sqrt{2}} y^3 \, dy = 2\pi \left. \frac{y^4}{4} \right|_0^{\sqrt{2}} = 2\pi \cdot \frac{(\sqrt{2})^4}{4} - 0 = \boxed{2\pi}$$

#18

$x = 2y - y^2 \qquad x = y \qquad x\text{-axis} \Rightarrow dy$



radius $= y$

ht $= (2y - y^2) - y$

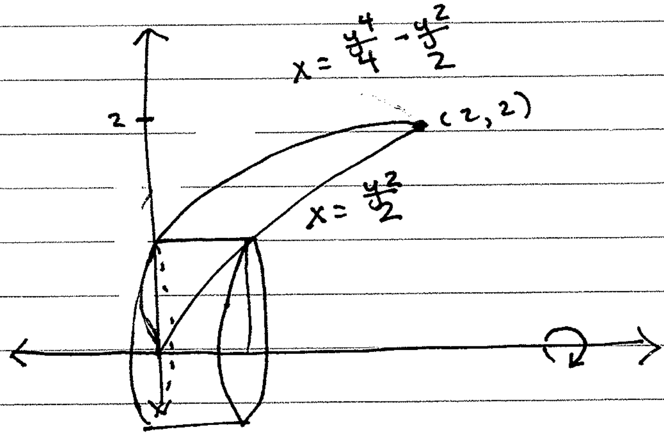
$$\int_0^1 2\pi \overset{r}{\overset{\downarrow}{y}} \underbrace{\left[(2y - y^2) - y\right]}_{\text{ht}} \, dy$$

$$= 2\pi \int_0^1 (y^2 - y^3)\, dy = \boxed{\frac{\pi}{6}}$$

24. a. x-axis $\quad \int_0^2 2\pi \overset{r}{y}\left[\frac{y^2}{2} - \left(\overset{ht}{\frac{y^4}{4}} - \frac{y^2}{2}\right)\right] dy = \frac{8\pi}{3}$

b. line $y = 2$

c. line $y = 5$

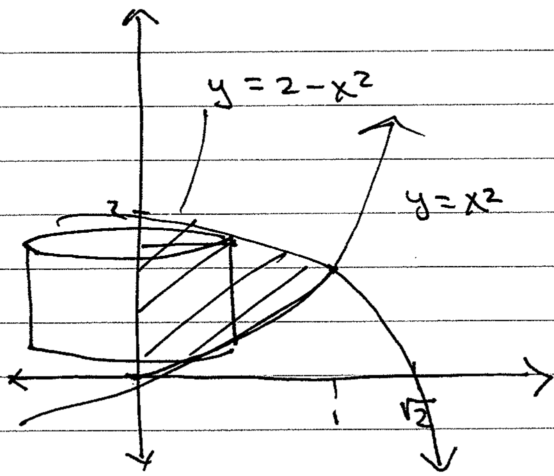


b. $$\int_0^2 2\pi\,\overset{r}{(2-y)}\left[\overset{h}{\frac{y^2}{2} - \left(\frac{y^4}{4} - \frac{y^2}{2}\right)}\right] dy = \frac{8\pi}{5}$$

c. $$\int_0^2 2\pi\,(5-y)\left[\frac{y^2}{2} - \left(\frac{y^4}{2} - \frac{y^2}{2}\right)\right] dy = 8\pi$$

insert $\frac{y^2}{2} -$

#10



$y = 2 - x^2, \quad y = x^2, \quad x = 0$

y-axis $\Rightarrow dx$

radius $= x$

ht $= (2 - x^2) - x^2$

$$\int_0^1 2\pi \overset{r}{x} \underbrace{\left[(2-x^2) - x^2\right]}_{ht} dx = \boxed{\pi}$$