

1. Evaluate the integrals:

$$(a) \int \frac{e^{\sqrt{t}}}{\sqrt{t}} dt \quad (b) \int \frac{\tan(\theta)d\theta}{2\sec(\theta)+1} \quad (c) \int_0^{1/\sqrt{2}} 2x \sin^{-1}(x^2) dx$$

$$(d) \int \frac{y dy}{y^2 - 2y - 3} \quad (e) \int \frac{(1-x^2)^{1/2}}{x^4} dx \quad (f) \int \frac{dx}{x^2\sqrt{x^2+1}}$$

2. Find the antiderivative of $y = \frac{1}{x - \sqrt{x}}$.

3. Find the centroid of the metal plate bounded by $y = x^2 e^x$, the x -axis and the line $x = 1$.

4. Solve the initial value problem: $(x+1)\frac{dy}{dx} = y^2 + 1$, $y(0) = \pi/4$

5. Find the volume of the solid generated by rotating the region bounded by $y = \frac{2}{1+x^2}$, $x = 0$, $y = 0$ and $x = 1$ rotated about the x -axis using the disk method.

6. Find the volume of the solid generated by rotating the region bounded by $y = \frac{2}{1+x^2}$, $x = 0$, $y = 0$ and $x = 1$ rotated about the y -axis using the shell method.

7. Do the following integrals converge or diverge? Show your work by either solving the integral or by stating the comparison test used and explicitly showing your reasoning.

$$(a) \int_0^1 -\ln x dx \quad (b) \int_{\pi}^{\infty} \frac{1 + \sin x}{x^2} dx \quad (c) \int_1^{\infty} \frac{1}{e^x - x - 1} dx$$

8. Do the following sequences converge or diverge? Explicitly show your reasoning.

$$(a) a_n = \left(1 - \frac{3}{n^2}\right)^n \quad (b) a_n = \frac{\log_n e}{\log_n n} \text{ for } n \geq 2 \quad (c) a_n = \frac{1 + n(-1)^n}{n!}$$

$$(d) a_n = \frac{(-1)^n(n+1)}{n} \quad (e) a_n = n^n e^{-2n} \quad (f) a_n = \frac{1 + \ln(5n^2)}{3n}$$

9. Do the following series converge or diverge? Explain why by stating the appropriate test and showing your reasoning.

$$(a) \sum_{n=2}^{\infty} \frac{100}{n^{3/2}} \quad (b) \sum_{n=1}^{\infty} \left(\frac{3}{n}\right)^{1/n} \quad (c) \sum_{n=3}^{\infty} \frac{2}{(n-1)(n-2)}$$

$$(d) \sum_{n=1}^{\infty} \frac{(-1)^n 2^{n-1}}{3^{n+1}} \quad (e) \sum_{n=2}^{\infty} \frac{n}{\sqrt{n^2-1}} \quad (f) \sum_{n=2}^{\infty} \frac{1}{(\ln \pi)^n}$$

10. Prove the limit: $\lim_{n \rightarrow \infty} \sqrt[n]{n} = 1$

11. Prove the limit: $\lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n = e^x$