ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (01 for 1-2 PM, and 02 for 2-3 PM), (4) your recitation time and recitation instructor, and (5) the color of your exam. You must work all of the problems on the exam. Show ALL of your work in your bluebook and box in your final answers. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books and class notes are NOT permitted. A calculator and a one-page crib sheet are allowed. **Start each problem on a new page**

1. (20 points) Mark in your bluebook True or False for each of the following statements (Credit will be given only for the correct boxed answer).

   a) A series converges if its n^{th} term approaches zero as $n \to \infty$.

   b) If the n^{th} term of a series limits to 1 as $n \to \infty$ then the series diverges.

   c) The series $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{(3^n)}$ is an alternating series.

   d) The ratio test can be used to show that the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{10}}$ converges.

   e) The ratio test can be used to show that the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n!}$ converges.

2. (20 points) For each of the following series, determine which converge absolutely, which converge conditionally, and which diverge. (No credit will be given without a detailed solution.)

   a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \ln(n)}{\ln(n^n)}$

   b) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n^3)}{n^3}$

   c) $\displaystyle\sum_{n=1}^{\infty} \frac{5^n + 7^n}{6^n}$

   d) $\displaystyle\sum_{n=1}^{\infty} 6^{-n}$

(OVER)

3. (20 points) Find the interval of convergence for the following series. Here you do not need to check the endpoints. (No credit will be given without a detailed method.)

   a) $\displaystyle\sum_{n=1}^{\infty} (x+3)^n$

   b) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$

   c) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-1)^n}{n!}$

   d) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n \ln n}{2^n}$

4. (20 points) Consider the function $f(x) = \dfrac{1}{1+x}$.

   a) Find the Maclaurin series for $f(x)$.

   b) Suppose $f(x)$ is approximated by the first three non-zero terms in the power series in part (a). Call this polynomial approximation $P_3(x)$. Show how to estimate the magnitude of the remainder term, denoted $R_3(x)$.

   c) Calculate the true value of $f(0.1)$. Now calculate the value of $P_3(0.1)$. Finally, calculate the value of $R_3(0.1)$ and verify that the magnitude of the true error, $|f(0.1) - P_3(0.1)|$, is less than $|R_3(0.1)|$.

5. (20 points) Perform the following calculations.

   a) Find the first 4 nonzero terms of the Maclaurin series for $\sin(2x)$.

   b) Differentiate your result from part (a) to find the first 4 non-zero terms of the Maclaurin series for $\cos(2x)$.

   c) Verify your answer in part (b) by integrating the Maclaurin series for $\sin(2x)$, from part (a), and comparing it to your result in part (b).

Is this infomation written clearly on the front of your bluebook:

your name?

your instructor's name and the time of your class?

the color of your exam?