

APPM 1360 --- Exam 2

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (01 for 1-2 PM, and 02 for 2-3 PM), (4) your recitation time and recitation instructor, and (5) the color of your exam.

You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX** in your final answers. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books and class notes are NOT permitted. A calculator and a one-page crib sheet are allowed. Please start each problem on a **NEW** page.

1. (20 pts) Evaluate the expressions in parts (a) and (b). Simplify the expressions in parts (c) and (d).

$$1. \frac{d}{dx} \tanh^{-1}(\sqrt{2x})$$

$$2. \int \frac{1}{\sqrt{x}} \cosh(\sqrt{2x}) dx$$

$$3. \ln(\cosh x + \sinh x) + \ln(\cosh x - \sinh x)$$

$$4. \cosh(\ln(\cosh x)) + \sinh(\ln(\cosh x))$$

2. (30 pts) Evaluate the following integrals. If you fail to do the integral analytically, then a correct numerical evaluation is worth 2 pts.

$$1. \int \frac{\cos(x)}{2 + \sin(x)} dx$$

$$2. \int_0^{\pi} e^{2x} \sin(x) dx$$

$$3. \int_0^{\pi} \sin^2(x) \cos^2(x) dx$$

$$4. \int \frac{1}{\sqrt{x^2 + 4x + 13}} dx$$

3. (10 pts) Give the **FORM** of the expansion of the following fractions into partial fractions. **DO NOT** evaluate the unknown coefficients, but be sure you have **EVERY NEEDED** term, and **ONLY** those terms needed.

1. $\frac{2x + 2}{(x^2 + 1)(x - 1)^3}$
2. $\frac{3x^2 + 2x + 3}{(x + 1)(x - 2)(x + 4)}$

4. (20 pts) In parts (a), (b), and (c), determine whether the integrals converge or diverge. If the integral converges, find its value analytically. In part (d), answer the question and justify your answer fully.

1. $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx$

2. $\int_0^\infty \frac{x^2 + 3x + 7}{\sqrt{x^3 + 7x + 3}} dx$

3. $\int_0^1 \frac{-1}{(x-1)(x-2)} dx$

4. What can one conclude about the convergence, or divergence, of the integral $\int_3^\infty \frac{\ln x}{x^2} dx$ given that one knows that $\int_3^\infty \frac{1}{x} dx$ diverges and that $\int_3^\infty \frac{1}{x^2} dx$ converges. **Explain your answer fully.**

5. (20 pts) For each of the following sequences determine whether $\lim_{n \rightarrow \infty} a_n$ exists. **Explain your answer fully.** If the limit exists, evaluate it.

1. $a_n = \frac{a_{n-1}}{2}$, and $a_0 = 1$

2. $a_n = \frac{e^n + \sin(n)}{e^n}$

3. $a_n = \frac{1}{n} + \sin(n)$

4. $a_n = \frac{2n + 1}{1 - 3\sqrt{n}}$