You must work all of the problems on the exam. Show ALL of your work in your bluebook and box in your final answers. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books and class notes are NOT permitted. A calculator and a two-page crib sheet are allowed. **Please start each problem on a new page.**

1. (20 points) Radioactive strontium-90, with a half-life of 29 years, can cause bone cancer in humans. The substance is carried by acid rain, soaks into the ground, and is passed through the food chain. The radioactivity level in a particular field is estimated to be 5.2 times the safe level S.

   a) Calculate the decay constant k for strontium-90. (Be sure to include the units!)

   b) For approximately how many years will this field be unsafe?

2. (30 points) Do the following integrals converge or diverge? If they converge do them **BOTH** analytically and numerically. Write down the function you type into your calculator, in addition to the answer it gives, to get credit for the numerical part.

   a) $\displaystyle\int_0^{\sqrt{2}} \frac{x}{\sqrt{4-x^4}}\,dx$

   b) $\displaystyle\int_0^1 \frac{5x}{(x-2)(x+3)}\,dx$

   c) $\displaystyle\int_0^{\pi/6} \cos^3(3\theta)\,d\theta$

   d) $\displaystyle\int_0^\infty \frac{1}{(x-1)^3}\,dx$

   e) $\displaystyle\int_1^\infty \frac{\sqrt{1+\sin x}}{x}\,dx$

3. (25 points) Determine whether the following series converge absolutely, conditionally, or diverge. Indicate clearly **WHY** your answer is correct.

   a) $\displaystyle\sum_{n=1}^\infty \frac{(-1)^n\cos(n\pi)}{n}$ b) $\displaystyle\sum_{n=1}^\infty \frac{(-1)^n e^n}{n!}$

   c) $\displaystyle\sum_{n=1}^\infty \frac{3\ln(n)+2}{n^2+2n+1}$ d) $\displaystyle\sum_{n=0}^\infty \frac{n^n}{3^n}$

4. (20 points) Consider the following two curves, $r_1^2 = \sin 2\theta$ and $r_2^2 = \cos 2\theta$.

   a) Neatly, and to scale, plot the two curves.

   b) Determine all of the intersection points.

5. (25 points) For each of the following sequences determine whether $\lim_{n\to\infty} a_n$ exists. **Explain your answer fully**. If the limit exists, evaluate it.
   a) $a_n = \left(\frac{a_{n-1}}{3}\right)^2$, and $a_0 = 1$
   b) $a_n = n(2^{1/n} - 1)^2$
   c) $a_n = \left(\frac{n+5}{5}\right)^{n/5}$
   d) $a_n = (10n)^{1/n}$

6. (25 points) It is known that $\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$. Use this fact to perform the following calculations.
   a) Calculate the Maclaurin series for $\tan^{-1}x$.
   b) Determine the radius of convergence for this series.
   c) We may evaluate the resulting Maclaurin series at $x = 1$ to calculate the value of $\pi/4$. How many non-zero terms are needed to calculate $\pi/4$ to within 10^{-1}?
   d) Numerically estimate $\pi/4$ by using the predicted number of terms in the Maclaurin series from part (c).
   e) Verify that your answer in part (d) is indeed within 10^{-1} of the correct value of $\pi/4$.

7. (30 points) Consider the following two curves, $r_1 = \sqrt{e^{\pi}}$ and $r_2 = e^{\theta}$, for $\theta > 0$.
   a) Neatly, and to scale, plot the two curves.
   b) Determine all of the intersection points.
   c) Calculate the arc length of curve r_2 that is **inside** curve r_1.
   d) Determine the area bounded by the x-axis, the y-axis and curve r_2.
   e) Determine the surface area of revolution that results from rotating curve r_2 (for $0 \le \theta \le \frac{\pi}{2}$) about the y-axis.
   f) Calculate the slope $\frac{dy}{dx}$ of **each curve** at all of the intersection points.

8. (25 points) For each picture below (a)-(d), find the equation or equations that give the picture. Your answer needs no explanation, just a number or numbers for (a)-(d).
   1) $r(\theta) = (\sin\theta)^{\sin 3\theta} \quad 0 \le \theta \le \pi$
   2) $x(t) = 3\cos(t), \quad y(t) = 2\sin(t) \quad 0 \le t < 2\pi$
   3) $\frac{x^2}{9} - \frac{y^2}{4} = 1$
   4) $r(\theta) = \sin 3\theta \quad 0 \le \theta < 2\pi$
   5) $x(t) = \sqrt{t}, \ y(t) = 3t$, for $t > 0$
   6) $x(t) = 3\cosh(t), \ y(t) = 2\sinh(t) \quad -\infty < t < \infty$
   7) $x = 3y^2$
   8) $r(\theta) = \cos 3\theta \quad 0 \le \theta < 2\pi$