APPM 1360 Due April 29, 1996

Calculator Problems — Set 2

For each of the following problems, use your calculator to plot the curves. Graph the results on paper and clearly label your axis and tick marks. Indicate appropriate values of t on the curves if they are parametric.

Be sure your calculator is in radians mode!

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1. The following curves should look like butterflies.
   a) $r = \exp(\cos\theta) - 2\cos(4\theta)$ for $0 \le \theta \le 2\pi$.
   b) $r = \exp(\cos\theta) - 2\cos(4\theta) + \sin^5\left(\frac{\theta}{12}\right)$ for $0 \le \theta \le 24\pi$.

2. The parametric curve $x = \cos(at)$ and $y = \sin(bt)$ is known as a Lissajous curve. Plot this for $a = 3$ and $b = 5$. These functions played an important role in the science fiction movie industry during the 1950's and 1960's.

3. The parametric curve $x = a\cos(t) + b\cos\left(\frac{at}{2}\right)$ and $y = a\sin(t) + b\sin\left(\frac{at}{2}\right)$ is known as a Hypotrochoid. Plot this for $a = 8$ and $b = 5$.

4. The curves defined by $x = a\cos(t) + b\cos(ct)$ and $y = a\sin(t) + b\sin(ct)$ should remind you of "Spirograph" patterns. Plot these curves for the following values:
   a) $a = 5$, $b = 10$, and $c = 2$
   a) $a = 5$, $b = 10$, and $c = 4$
   b) $a = 12$, $b = 6$, and $c = 2$
   b) $a = 12$, $b = 6$, and $c = 4$

5. Different parametric representations may give different segments of the same curve. Graph the following curves in the xy plane and discuss the similarities and differences.
   a) $x = -2 + t^2$ and $y = 1 + 2t^2$ for all real t
      $x = -2 + t$ and $y = 1 + 2t$ for all real t
   b) $x = t$ and $y = 1 - t$ for all real t
      $x = 1 - t^2$ and $y = t^2$ for all real t
      $x = \cos^2(t)$ and $y = \sin^2(t)$ for all real t
      $x = \ln(t) - t$ and $y = 1 + t - \ln(t)$ for $t > 0$

(OVER)

6. Suppose two Ferris wheels are next to each other. The motion of Sue on the first Ferris wheel is described by the parametric equations $x_1 = 20\cos\left(\frac{\pi t}{6}\right)$ and $y_1 = 20 + 20\sin\left(\frac{\pi t}{6}\right)$ for $0 \leq t$. Similarily, Bob's motion on the second Ferris wheel is described by $x_2 = 15 + 15\cos\left(\frac{\pi t}{4}\right)$ and $y_2 = 15 + 15\sin\left(\frac{\pi t}{4}\right)$.

   a) Plot the paths of the two Ferris wheels.

   b) Use the *trace* feature of your calculator to **estimate** how much time passes before **both** Bob **and** Sue are at one of the intersection points of the curves.

   c) Use the *trace* feature of your calculator to **estimate** the first time that Bob and Sue are separated by the maximum distance possible.

   d) Use the *trace* feature of your calculator to estimate the locations at which the curves intersect. Report the xy coordinates of these points, as well as the values of t.

   e) Use the *calc* feature of your calculator to estimate the slope dy/dx of each curve at the intersection point which is the highest above the ground.

   f) Finally, use the *fnInt* function on your calculator to estimate the arc length of the portion of the smaller Ferris wheel that overlaps the larger Ferris wheel.