APPM 1360 Spring 1996

EXAM #3

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) lecture section (4) your recitation time and recitation instructor, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books and class notes are NOT permitted. A calculator and a one-page crib sheet are allowed. Please start each new problem on a new page.

1. (15 points) In your bluebook either write the entire word TRUE or FALSE for each of the following statements. (Credit will only be given for the correct boxed answer. There is no partial credit for problems 1(a) through (e) and no work need be shown.)

   (a) If $\lim_{n\to\infty} a_n = L$, where L is any finite value, the sequence converges.

   (b) $\sum_{n=0}^{\infty} 3(e^x - 3)^{-n}$ is a power series.

   (c) $\sum_{n=0}^{\infty} 3(e^x - 3)^{-n}$ is a geometric series.

   (d) If $a_n = \dfrac{1}{(n+1)(n+2)}$, then the series $\sum_{n=1}^{\infty} a_n$ converges to 1.

   (e) The series $\sum_{n=0}^{\infty} \dfrac{(10,000,000)^n}{n!}$ diverges.

2. (18 points) For each of the following series, determine which converge absolutely, which converge conditionally and which diverge. (No credit will be given without a detailed solution.)

   (a) $\sum_{n=1}^{\infty} \dfrac{(-1)^n}{2n^2 + \ln n}$

   (b) $\sum_{n=1}^{\infty} n\left(\dfrac{3}{4}\right)^n$

   (c) $\sum_{n=2}^{\infty} \dfrac{1}{n \ln(2n)}$

3. (21 points) Find the interval of convergence for each of the following series. Be sure to check the endpoints and describe the type of convergence there. (No credit will be given without a detailed solution.)

   (a) $\sum_{n=1}^{\infty} (\ln x)^n$

   (b) $\sum_{n=1}^{\infty} \dfrac{(x-3)^n}{3^n(n+2)}$

   (c) $\sum_{n=1}^{\infty} n^n x^n$

4. (21 points) Suppose $a_0 = 3$ and that $a_n = \left(\frac{x}{2}\right) a_{n-1}$ for $n \geq 1$.

   (a) List the first 4 terms of the *sequence*.

   (b) Find an explicit formula for a_n.

   (c) Find the limit of the terms in the *sequence* when $x = -\frac{1}{4}$.

   (d) Estimate the error if you use the sum of the first three terms of the *sequence* to approximate $\sum_{n=0}^{\infty} a_n$ when $x = -\frac{1}{4}$.

   (e) Find the limit of the *sequence of partial sums* when $x = -\frac{1}{4}$.

5. (25 points) We would like to evaluate $\sum_{n=1}^{\infty} (-1)^n \frac{n}{2^n}$. The following adventure will lead you through this calculation — have confidence and follow the directions carefully!

   (a) From its definition, calculate the Maclaurin series for $\frac{1}{1+x}$. (Your result should have the interval of convergence $-1 < x < 1$ but you need not verify this.) Set $\frac{1}{1+x}$ equal to its Maclaurin series.

   (b) Differentiate, with respect to x, both sides of your result from part (a).

   (c) Now, multiply both sides of your result in part (b) by x. Substitute $x = \frac{1}{2}$ into both sides to find the exact value of $\sum_{n=1}^{\infty} (-1)^n \frac{n}{2^n}$.

   (d) "Verify"your answer in (c) by using your calculator to estimate the sum $\sum_{n=1}^{\infty} (-1)^n \frac{n}{2^n}$. Clearly write down the keystrokes you use.