APPM 1360 FALL 1996

Exam 3

November 13, 1996, 7:30 - 9:00 p.m.

ON THE EXAM BOOKLET PLEASE: (1) Write your name, (2) Student ID, (3) Lecture Number, and (4) Recitation Instructor.

Show all work. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit.

I. (25 pts.) Determine whether the following series converge absolutely, converge conditionally, or diverge. You must show all of your work.

A. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(1+\ln n)}$

B. $\displaystyle\sum_{n=1}^{\infty} (-\sqrt{2})^n$

C. $\displaystyle\sum_{n=2}^{\infty} (-1)^{n+1} \frac{1}{n \ln n}$

II. (30 pts.) Find the interval of convergence for the following series. Be sure to check the endpoints (if there are any) and state whether they converge absolutely, conditionally, or diverge. (For two points of extra credit each, state the functions to which the series converge.)

A. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+3}}{(2n+1)!}$

B. $\displaystyle\sum_{n=1}^{\infty} n^n (x-\pi)^n$

C. $\displaystyle\sum_{n=1}^{\infty} (1+\frac{1}{n})^n x^n$

III. (20 pts.) Compute the series for $\tan^{-1}(x)$ by following the steps outlined below.

A. Write down a geometric series which has the following sum $\dfrac{1}{1+x^2}$.

B. Integrate it to find the series for $\tan^{-1}(x)$. What must the constant of integration be?

IV. (25 pts.) Let $a_0 = 1$ and define $a_{n+1} = (\ln x) a_n$.

A. Write the first four elements of this sequence. Give the formula for the general term of this sequence.

B. For what values of x does the sequence converge?

C. For what values of x does the series $\displaystyle\sum_{n=0}^{\infty} a_n$ converge? Don't forget to check the endpoints for absolute or conditional convergence or divergence.

D. To what function does the series converge?