

APPM 1360 - Spring 1998

Exam # 3

On the front of your bluebook, print your name, student number, the name of your instructor, the time of your class, the color of your exam and a grading table. There are 5 questions. It is very important that you show all your work in the bluebook. Box in your answers, if possible. Calculators are allowed, but crib sheets are not. You will be provided with a printed table of most frequently used integrals, limits and series.

1. (20 Points) For each of the following series find the values of x for which the series converges absolutely

a. $\sum_{n=1}^{\infty} \frac{n}{n+2} x^n$

b. $\sum_{n=1}^{\infty} \frac{1}{n^2} (x+1)^n$

2. (20 Points) The Maclaurin series for $\tan^{-1}(x)$ is

$$\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$

- a. Find the Maclaurin series for $x \tan^{-1}(x)$.
- b. For what range of x does the series for $x \tan^{-1}(x)$ converge absolutely? Where does it converge conditionally? Where does it diverge?
3. (15 Points) Solve the following initial value problem

$$x y' - y = 1 + x^2, \quad y(0) = 1$$

4. (25 Points)

- a. Use Taylor's formula with $n = 2$ to find the quadratic approximation of $f(x) = (1+x)^{5/2}$, valid near $x = 0$.
- b. Write the formula for the error, $R_2(x)$, in approximating $f(x)$ by its quadratic approximation near $x = 0$. (Make certain that all the terms in your formula are defined.)
- c. For approximately what values of x in the interval $[0, 1]$ will the error in the quadratic approximation be less than $\frac{1}{100}$?

5. (20 Points)

- a. Sketch the curve in the xy -plane defined parametrically by

$$x = 2 \cos t, \quad y = \sin t$$

with $0 \leq t \leq 2\pi$.

- b. Find the point (or points) on the curve closest to the point $(3/4, 0)$.
c. Find the point (or points) on the curve farthest from the point $(3/4, 0)$.