APPM 1360 **EXAM #3** **FALL 1999**

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) your instructor's name, (4) your recitation number, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes and calculators are NOT permitted. A one-page crib sheet is allowed. Please start each new problem on a new page of the bluebook.

1. (25 points) Determine whether the following series converge or diverge. Give reasons for your answers.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n+5}{\sqrt{n^6+3n^2+7}}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1+\cos(n)}{n^2}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n n^3}{n!}$

   (d) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln(n)}$

2. (20 points) Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}n}{n^2+4}$ is absolutely convergent, conditionally convergent or divergent. Give reasons for your answers

3. (25 points) For the power series $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{3^n(n^3+6)}$ determine:

   (a) The interval of convergence

   (b) Where the power series is absolutely convergent

   (c) Where the power series is conditionally convergent

4. (15 points) Consider the function $f(x) = (1+x)^{\frac{1}{2}}$. If you were to calculate the third derivative with respect to x, you would find that $f'''(x) = -\frac{3}{8}(1+x)^{\frac{1}{2}-3}$. Estimate the error in the approximation $f(-\frac{1}{2}) \approx P_3(-\frac{1}{2})$, where $P_3(x)$ is computed with a center of $a = 0$. Note that you have just estimated the error associated with approximating $1/\sqrt{2}$ by $P_3(-\frac{1}{2})$.

5. (15 points) Consider the following steps for evaluating $\displaystyle\int_0^{\frac{1}{2}} \sin(x^3)\,dx$.

   (a) Compute the MacLaurin series for $\sin(x^3)$.

   (b) Using the MacLaurin series from part (a), find $\displaystyle\int_0^{\frac{1}{2}} \sin(x^3)\,dx$.

   (c) Estimate how accurately the first four nonzero terms of the series approximate the integral's value. You may leave your answer in terms of factorials, etc.