APPM 1360 **OPTIONAL EXAM #2** **SPRING 2000**

May 3, 2000

1. (30 points) Compute the following integrals. If the integral is improper, determine whether it converges or diverges, and if possible, determine its value. Be sure to show all your work to receive full credit.

   (a) $\int e^{2x} \cos 3x \, dx$

   (b) $\int_0^1 \frac{dx}{(4-x^2)^{3/2}}$

   (c) $\int_3^\infty \frac{\ln x}{x^2} \, dx$

   (d) $\int \sin^{-1} x \, dx$

   (e) $\int_0^9 \frac{dx}{(x-3)^2}$

   (f) $\int \frac{3x^2+2x+5}{(x-1)^2(x^2+4)} \, dx$

   (g) $\int_1^3 x^2 \ln(x) \, dx$

   (h) $\int x(\ln x)^2 \, dx$

   (i) $\int \frac{dx}{x(3\sqrt{x}+1)} \, dx$

   (j) $\int \frac{1}{x^2\sqrt{x^2+9}} \, dx$

   (k) $\int_3^\infty \frac{1}{\ln x} \, dx$

   (l) $\int_3^\infty \frac{x}{\ln x} \, dx$

   (m) $\int_0^\infty \frac{\ln x}{x} \, dx$

   (n) $\int_0^1 \ln(x) \, dx$

   (o) $\int_0^1 x \ln(x) \, dx$

   (p) $\int_1^\infty x^2 e^{-x} \, dx$

   (q) $\int_1^\infty \frac{1}{x^2}\sqrt{1-\frac{1}{x^6}} \, dx$

   (r) $\int_1^\infty = \frac{dx}{\sqrt{x+x^4}}$

   (s) $\int_0^\infty \left(\frac{1}{x^3}+\frac{1}{x}\right) dx$

2. (20 points) Determine whether the following sequences converge or diverge. If the sequence converges, find the limit. Be sure to show your work.

   (a) $a_n = \left(\frac{1+\ln n}{\ln n}\right) \sin\left(\frac{n\pi}{2}\right)$ for $n \geq 2$

   (b) $a_n = \frac{e^{n+2}}{ne^n}$

   (c) $a_n = \left(\frac{1}{n^2}\right)^{n^2}$

   (d) $a_n = \frac{x^n}{n}$ for $-1 < x < 1$

   (e) $a_n = nx^n$ for $-1 < x < 1$

   (f) $a_n = 1 + (-1)^n \left(\frac{1}{n}\right)^n$

   (g) $a_n = \left(\frac{1}{n}\right)^n$

   (h) $a_n = \left(n\right)^{\frac{1}{n}}$

   (i) $a_n = \frac{\log_n e}{\log_n n}$ for $n \geq 2$

   (j) $a_n = \frac{a_{n-1}}{n}$ if $a_1 = 1$

3. (20 points) State whether each of the following statements is true (T) or false (F). If it is false, give a counterexample (i.e., an example that shows the statement is not necessarily true).

   (a) If $a_n \to 0$ as $n \to \infty$, then $\sum_{n=1}^{\infty} a_n$ converges.

   (b) If $\sum_{n=1}^{\infty} a_n$ converges, then $a_n \to 0$ as $n \to \infty$.

(c) If $a_n \geq 0$, $c_n \geq 0$, $c_n \leq a_n$ for all $n \geq 1$, and $\sum_{n=1}^{\infty} c_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.

(d) If $\lim_{N\to\infty} \left(\sum_{n=1}^{N} a_n \right)$ exists and is finite, then $\sum_{n=1}^{\infty} a_n$ converges.

4. (30 points) Evaluate each of the following.

(a) $\sum_{n=0}^{\infty} \frac{2^{n+1}}{5^n}$

(b) $\sum_{n=1}^{\infty} \frac{5}{(5n-4)(5n+1)}$

(c) $\lim_{n\to\infty} \left(\frac{n}{n+1} \right)^n$

(d) $\sum_{n=1}^{\infty} = \frac{2n+1}{2n}$

(e) $\sum_{n=1}^{\infty} \frac{2^{2n}}{4^{2n+2}}$

(f) $\sum_{n=3}^{\infty} \frac{1}{\ln(\ln x)}$

(g) $\sum_{n=3}^{\infty} \frac{\ln n}{n}$

(h) $\sum_{n=1}^{\infty} \frac{2^n \cos(n\pi)}{4^{n+2}}$

5. (20 points) Let R be the region in the first quadrant that is bounded above by the curve $y = \ln x$ and on the right by the line $x = e$. Sketch the region R and calculate its area.

6. (15 points) For your birthday, you treat yourself to a bungee-jump. When you are dropped, the initial distance you fall is 100 feet. Each time the bungee cord pulls you back, you only go 80% of the way back up. Find the total distance you travel up and down.