APPM 1360 **Exam #3** April 19, 2000

On the front of your bluebook, print
your name *the name of your instructor*
the time of your lecture *a grading list*

There are 5 questions, on two sides of this sheet. Answer all parts of all 5 questions. Show all your work in your bluebook. Box in your answers. **No** calculators are allowed. A set of frequently used Maclaurin series is provided on the other side of this sheet.

1. (10 points) Find the value of ***b*** so that: $1 + e^{b} + e^{2b} + e^{3b} + ... = 9.$

2. (20 points) Consider the series $\sum_{n=1}^{\infty} a_n$, $a_n = \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}$.

a) Find the first three terms of the sequence $\{a_n\}$.
b) Find the first three partial sums: s_1, s_2, s_3.
c) Find a simple expression for the n^{th} partial sum, s_n.
d) Does the *sequence* $\{a_n\}$ converge? If so, what is its limit?
e) Does the *sequence* $\{s_n\}$ converge? If so, what is its limit?
f) Does the *series* $\sum_{n=1}^{\infty} a_n$ converge? If so, what is its limit?

3. (20 points)

a) Does the series $\sum_{n=1}^{\infty} (1 + \frac{1}{n})^n$ converge or diverge? Justify your answer.

b) Find the radius of convergence of the series $\sum_{n=1}^{\infty} (1 + \frac{1}{n})^n (x+2)^n$.

c) For what values of x does the series in part (b) converge absolutely? Where does it converge conditionally? Where does it diverge? Justify your answers.

(*over*)

4. (25 points)

a) For $f(x) = e^x$, write down $P_3(x)$, the cubic polynomial approximation to e^x, valid near $x = 0$.

b) Evaluate $P_3(x)$ at $x = 1$, to find an approximate value of e.

c) What is the smallest (N) such that the error in approximating e by $P_N(x)$ at $x = 1$ is no larger than 0.01 in magnitdue. Show your work. [You may use the fact that $e < 3$.]

5. (25 points)

a) Find $y(x)$ so that $(1-x)\dfrac{dy}{dx} - y = 0, \qquad y(0) = 2.$

b) For $y(x)$ defined in (a), evaluate $\dfrac{dy}{dx}(0)$.

Frequently used Maclaurin Series

1. $\dfrac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \ldots + x^n + \ldots = \sum_{n=0}^{\infty} x^n, \quad |x| < 1$

2. $\dfrac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 + \ldots + (-x)^n + \ldots = \sum_{n=0}^{\infty} (-1)^n x^n, \quad |x| < 1$

3. $e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots + \dfrac{x^n}{n!} + \ldots = \sum_{n=0}^{\infty} \dfrac{x^n}{n!}, \quad |x| < \infty$

4. $\sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} - \ldots + (-1)^n \dfrac{x^{2n+1}}{(2n+1)!} + \ldots = \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{2n+1}}{(2n+1)!}, \quad |x| < \infty$

5. $\cos x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} - \ldots + (-1)^n \dfrac{x^{2n}}{(2n)!} + \ldots = \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{2n}}{(2n)!}, \quad |x| < \infty$

6. $\ln(1+x) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \ldots + (-1)^{n-1} \dfrac{x^n}{n} + \ldots = \sum_{n=1}^{\infty} \dfrac{(-1)^{n-1} x^n}{n}, \quad -1 < x \le 1$

7. $\ln\left(\dfrac{1+x}{1-x}\right) = 2\left(x + \dfrac{x^3}{3} + \dfrac{x^5}{5} + \ldots + \dfrac{x^{2n+1}}{2n+1} + \ldots\right) = 2\sum_{n=0}^{\infty} \dfrac{x^{2n+1}}{2n+1}, \quad |x| < 1$

8. $\tan^{-1} x = x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \ldots + (-1)^n \dfrac{x^{2n+1}}{2n+1} + \ldots = \sum_{n=0}^{\infty} \dfrac{(-1)^n x^{2n+1}}{2n+1}, \quad |x| \le 1$

9. Binomial series

$$(1+x)^m = 1 + mx + \frac{m(m-1)}{2!}x^2 + \frac{m(m-1)(m-2)}{3!}x^3 + \ldots + \frac{m(m-1)\cdots(m-k+1)}{k!}x^k + \ldots$$

$$= 1 + \sum_{k=1}^{\infty} \binom{m}{k} x^k, \quad |x| < 1$$