APPM 1360 **EXAM #1** **SPRING 2001**

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your student ID number, (3) your instructor's name, (4) your recitation number, and (5) a grading table. You must work all of the problems on the exam. Show ALL of your work in your bluebook and **BOX IN YOUR FINAL ANSWERS**. A correct answer with no relevant work may receive no credit, while an incorrect answer accompanied by some correct work may receive partial credit. Text books, class notes and calculators are NOT permitted. Please start each new problem on a new page of the bluebook.

1. (15 points) Compute the derivatives of the following functions and simplify your answers.

   (a) $y = \tanh^{-1}(\ln x)$

   (b) $y = 2^{\cos x}$

   (c) $y = \sinh^2(\sinh x)$

2. (20 points) Evaluate the following integrals and simplify your answers.

   (a) $\displaystyle\int x \ln(x)\, dx$

   (b) $\displaystyle\int \frac{dx}{x^2+8x+17}$

   (c) $\displaystyle\int e^x \sin(2x)\, dx$

   (d) $\displaystyle\int \frac{dx}{(1-x^2)^{3/2}}$

3. (20 points)

   (a) Evaluate the following integral: $\displaystyle\int \frac{3x^2+x+1}{(x-1)(x^2+4)}\, dx.$

   (b) Set up, but **do not evaluate** the coefficients, in the partial fraction decomposition of $\dfrac{x^5+x^3+9x^2-4x+7}{(x^2+1)^2(x-4)^3(x+5)}$.

4. (20 points) Consider the region in the first quadrant bounded above by the line $y = 1$, below by $y = \cos x$, and on the right by $x = \dfrac{\pi}{2}$.

   (a) Find the volume of the solid generated by revolving this region about the y-axis.

   (b) Find the volume of the solid generated by revolving this region about the line $y = 1$.

5. (15 points) Consider a circle of radius R centered on the origin. Set up, but **do not evaluate**, the integral required to determine the arc length which is in the first quadrant.

6. (10 points) Solve the following initial value problem:

$$\begin{aligned} \frac{dy}{dx} &= -e^{\cos x} e^{-y} \sin x \\ y(0) &= 1\,. \end{aligned}$$

7. (Extra credit 10 points) Consider the region in the first quadrant bounded above by the line $y = 1$, below by $y = \cos x$, and on the right by $x = \dfrac{\pi}{2}$. (This is the region from problem 4.) Find the volume of the solid generated by revolving this region about the line $x = \dfrac{\pi}{2}$.

APPM 1360 **Exam #1 Formula Sheet** **SPRING 2001**

A short table of integrals. In the following, $a \neq 0$.

(a) $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}(u/a) + C$ for $u^2 < a^2$

(b) $\displaystyle\int \frac{du}{a^2+u^2} = (1/a)\tan^{-1}(u/a) + C$

(c) $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} = (1/a)\sec^{-1}|u/a| + C$ for $u^2 > a^2 + C$

(d) $\displaystyle\int \frac{du}{\sqrt{a^2+u^2}} = \sinh^{-1}(u/a) + C$ for $a > 0$

(e) $\displaystyle\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}(u/a) + C$ for $u > a > 0$

(f) $\displaystyle\int \frac{du}{a^2-u^2} = \begin{cases} (1/a)\tanh^{-1}(u/a) + C & \text{if } u^2 < a^2 \\ (1/a)\coth^{-1}(u/a) + C & \text{if } u^2 > a^2 \end{cases}$

(g) $\displaystyle\int \frac{du}{u\sqrt{a^2-u^2}} = -(1/a)\,\text{sech}^{-1}(u/a) + C$ for $0 < u < a$

(h) $\displaystyle\int \frac{du}{u\sqrt{a^2+u^2}} = -(1/a)\,\text{csch}^{-1}|u/a| + C$ for $u \neq 0$